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# Post-Keynesian vignettes on secular stagnation: From labor suppression to natural growth*

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**Abstract**

The stylized facts of neoliberalism include a decline in steady state rate of growth and labor share. Recent classical-Keynesian literature sees the latter as a cause for the former. A crucial element is the distinction between short and long run. The business cycle is profit-led and profit-squeeze, but the steady state features a wage led natural rate of growth. This paper presents simple macroeconomic models in this vein. Our starting point is to assume an adverse shock to real wage bargaining, which across all models depresses the labor share. We consider (i) a two-dimensional model in income-capital ratio and labor share, a (ii) three-dimensional model that adds the employment rate as state variable, and a (iii) four-dimensional model that furthermore endogenizes the savings propensity. Key results are that model (i) predicts an increase (decrease) in the warranted (natural) rate of growth, and thus does not generate balanced growth; (ii) resolves this problem and predicts stagnation in steady state, but implies a long run paradox of thrift; and (iii) allows for contextualization vis-à-vis the utilization controversy.

**Keywords**: Goodwin theory; labor suppression; secular stagnation.
**JEL Classification**: E12, E25, E32, J50.

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## 1 Introduction

This paper outlines post-Keynesian theory that renders capital's onslaught on labor during the neoliberal era a causal factor for stagnationary tendencies. The link from inequality to stagnation is often seen in the crucial role of wages to fund consumption demand. In contrast to this view, we consider the deleterious effect of labor suppression on the natural rate of growth. In other words, low wages constrain growth in the long run, whereas in the short run economic activity might benefit therefrom. Our argument is, in a nutshell, that neoliberal policymakers and their corporate funders appear to have concluded that "in the long run, we're all dead:" feed the monster, now, has been the mantra.

In all of it, the labor share of income takes center stage. It is the portion of total income that labor receives for (paid) work. It is also conceptually equivalent to the ratio of real wage to labor productivity, and as such provides a key entry point to debates about the relationship between earned income and technology. The labor share of income has been declining since roughly 1980, and this trend has accelerated after 2000. Hence, the growth of average real wages significantly lags behind the growth of average labor productivity. At the same time, this rate of labor productivity growth has also decreased: if there is a robot revolution, it yet has to manifest in aggregate productivity statistics.

Indeed, the slowdown in labor productivity growth precedes the Great Recession by more than a decade, and runaway technological improvements are definitely not the cause of the decline in the labor share. Further, each of the last three recoveries (pre-Covid) have been termed "jobless." Apparently, macroeconomic policy—monetary, long unconventional but ineffective, but also fiscal policy for the purpose of stabilization, and public expenditures targeting all of the physical and socio-economic infrastructure for the long run—has been unable to generate sufficient growth, employment or inflation.

We present here standard neo-Goodwinian growth theory that formalizes key elements of this narrative. This theory is Keynesian-Kaleckian in that it features an independent expenditure function and hence demand-driven growth, and it is Goodwinian in the sense that it includes real wage bargaining, and emphasizes the important role of labor constraints to growth. Crucially, labor productivity growth is endogenous to the labor share, à la induced technical change. Moreover, and in sharp contrast to classical models, the steady state labor share responds to parameters that describe labor market institutions. In combination, this implies that an adverse shock to real wage bargaining depresses the labor share *and* the steady state growth rate.

We consider in Section 3 a two-dimensional model in income-capital ratio and labor share, in Section 4 a three-dimensional model that adds the employment rate as state variable, and in Section 5 a four-dimensional model that furthermore endogenizes the savings propensity. Key results are that the first model predicts an increase in the warranted rate of growth, but a decrease in the natural rate of growth. The divergence arises since the model does not specify balanced growth in steady state. The three-dimensional model resolves the issue and predicts stagnation in steady state. Like the two-dimensional model, it implies a long run paradox of thrift. Lastly, the four-dimensional model allows for contextualization vis-à-vis the utilization controversy: the endogenous savings propensity forces convergence to a desired rate of utilization. It also renders the paradox of thrift inoperable in the long run, since a demand shock is accommodated by an *increase* in the savings propensity. Still, the steady state rate of growth remains endogenous to distributive and demand shocks, and the model thus retains important Keynesian features.

Moreover, the labor-constrained models of Sections 4 and 5 indicate that demographic change (i.e., ageing of the workforce and decreases in fertility) and an exhaustion of impactful innovations (trains and air conditioning) are *not* by themselves plausible drivers of stagnation and inequality. Instead, weakened labor market institutions lead to a lower labor share, which subsequently reduces pressures to innovate. Additionally, significant reductions in public investments—to the extent that this can be captured by the intercept of the independent expenditure function—complement and amplify these effects. Stagnation is thus not the result of pure supply side changes, but much rather the result of social conflict, *won by capital*.

In the next section, we motivate our approach with an overview of relevant data.

## 2 Stylized facts of the neoliberal era

In recent decades, the US has been marked by pronounced trends in important macroeconomic variables. Key among these are the decline in the steady state growth rate and the decline in the labor share of income. Further, the income-capital ratio has decreased, the employment rate has risen or at least remained at elevated levels, and the components of natural growth—growth rates of labor productivity and labor force—have also declined.

All of these are trend changes that interact with persistent and relatively regular oscillations at business cycle frequency. These include counter-clockwise cycles in employment rate and labor share, income-capital ratio and labor share and moreover income-capital ratio and employment rate. Put differently, the income-capital ratio—proxy for aggregate demand at business cycle frequency—leads the labor share and also the employment rate. Throughout this paper, we emphasize theory that places aggregate demand center stage.

Here, we review these data for the post-war US. We focus on long run changes rather than cyclical patterns. For a discussion of the latter, see Zipperer and Skott (2011) and Barrales et al. (2021b). Our objective is to motivate subsequent theory. For similar efforts, see Foley et al. (2019, Ch. 2), Petach and Tavani (2020, Fig. 1), Petach and Tavani (2021, Fig. 1) and Setterfield (2021, Table 2).

To begin, we introduce notation. Harrod's three growth rates—equal in steady state—provide a convenient framework:

$$g = s_\pi(1-\psi)u = a + n, \tag{2.1}$$

where $g = I/K = \hat{Y}$ is the realized rate of growth (with $K$ non-depreciating capital stock, $I = \dot{K}$ and $Y$ the level of real GDP; further below, we use $\hat{Y} = h$), the second term is the warranted rate of growth $g^w$, and the third is the natural rate of growth $g^*$.[1] $s_\pi$ is the savings propensity of capitalists. $\pi = 1 - \psi = 1 - wL/PY = 1 - \Omega/A$ is the profit share, and $\psi$ the labor share. The growth rate of labor share is equal to the difference between the growth rates of real wages $\hat{\Omega} = \omega$ and labor productivity $\hat{A} = \hat{Y} - \hat{L} = a$. $u = \sigma U = (Y^*/K)(Y/Y^*) = Y/K$ is the income-capital ratio and the product of full capacity output to capital ratio $\sigma$ and utilization rate $U$. The growth rate of the labor force is $\hat{N} = n$, so that $g^* = a + n$.

Table 1 and Figure 1 provide an overview of key series. All of these series portray clear cyclical patterns, but our focus lies on trend changes. Indeed, and for our purposes, the centrally important insight is that the neoliberal era—also commonly and euphemistically labeled as the *Great Moderation*—differs quite clearly from the initial post-war decades—also, at least until 1973, often labeled as the *Golden Age*.

The growth rate of real GDP averages close to four percent per year, whereas post-1980 it declines to about two and a half percent per year. The same is true for the components of the natural rate of growth: the growth rates of output per hour worked and the labor force. Both average growth rates decrease significantly from the first period to the second. Though they sum to more than the growth rate of real GDP across both sub-samples, a significant majority of realized growth is accounted for by the growth rate of the effective labor force.

The growth rate of the real wage shows an astounding drop, from an average of a bit more than two percent per year to barely registering as positive. During both periods, the average growth rate of output per hour is larger than that of real wages. However, the gap widens. The implication is that the labor share falls after 1980. Two points are worth emphasizing here. First, the measurement of the labor share is a contentious topic—but there is no controversy over the fact that it has declined precipitously since 1980.[2] Second, the dramatic decline in the labor share is driven predominantly by weakening of real wage growth. *Ceteris paribus*, the decline in the growth rate of labor

[1] Throughout, and as is standard, for any variable $x$, $\dot{x} = dx/dt$ is the time rate of change and $\hat{x} = \dot{x}/x$ is the proportional growth rate.

[2] Here we utilize the standard BLS "headline" index, which is available from FRED. See Mendieta-Muñoz et al. (2020, Section 2) and Barrales et al. (2021b, Section 3.2) for discussions on measurement issues, and endnote 15 in the latter paper for the source of the headline labor share in percentages.

productivity would have *increased* the labor share. The inability of labor to garner real wage growth even at the slower rate of labor productivity growth has to be seen as driven by changing labor market institutions.[3] For empirical evidence and critical discussions, see Mendieta-Muñoz et al. (2020) and Setterfield (2021). The patterns observed here serve as crucial motivation for the labor suppression shock imposed upon models in subsequent sections.

The income-capital ratio $u$ is hard to measure—but easier to assess than either the rate of utilization $U$ or the technical coefficient $\sigma$. The proxy we employ here captures only the corporate sector. Specifically, it is the ratio of corporate value added to current cost estimates of corporate fixed assets. We thus miss non-corporate activity. The question arises whether public capital should be accounted for, too. Other proxies commonly used for the rate of utilization are detrended real output series, or the ratio of real GDP to the CBO's estimate of potential output. The former is stationary by construction; the latter shows a similar though slightly less pronounced decline in the neoliberal era as the income-capital ratio in Panel (b).[4]

The employment rate shows a mixed picture. We include two different measures in Panel (d) to illustrate. The first is the remainder to one of the civilian unemployment rate. This is the solid line in the chart; it is a commonly used measure of the employment rate in the post-Keynesian empirical literature. The second proxy is the prime age employment rate, shown as a dashed line. It is the employment-population ratio for 25–54 year olds. The average of the solid line has decreased slightly in the neoliberal era, the average of the second has increased. The increase in the overall employment rate is driven by the female participation rate, wheres the unemployment rate takes participation into account. Either series could be justified as preferable.

Last but not least, Panel (h) shows selected public expenditures as a share of nominal GDP. We draw freely on Petach and Tavani (2021, Fig. 1), but add the three series displayed there. The key issue for our purposes is that *public deficits are a poor proxy of stagnationary impulses of policy*: for about two decades, public net borrowing has been persistently high. It, however, has also been poorly targeted, with wasteful giveaways to both large corporations and the top sliver of the personal income distribution. The dramatic decline of these selected public expenditures relative to GDP illustrates that—behind the veil of high public debts—austerity reigns. We draw on this insight to motivate the adverse shock to demand imposed upon models below.

In summary, we see two shocks as centrally important, and persistent: an adverse shock to labor's bargaining, and an adverse shock to aggregate demand. Both appear

[3] Structural change—a shift of employments towards activities with relatively low productivity and low pay—is likely fueling adverse changes to labor's bargaining power; see also footnote 14.

[4] The decline is less pronounced because real potential GDP (GDPPOT in FRED) has repeatedly been revised downward. In this sense, the estimate of potential growth is utterly dependent on actual growth, despite myriad and strong theoretical and empirical assumptions about the supply side made in its construction.

driven by a conservative bias in policy. These shocks are tasked to generate the stylized facts of the neoliberal era: $g^* \downarrow$, $u^* \downarrow$ and $\psi^* \downarrow$, whereas $e^*$ has increased or roughly held level. As will be seen, the mixed picture for the employment rate might emerge due to differential impacts of the two shocks.

## 3 A Keynesian model of the distributive cycle

This section presents a simple two-dimensional model with neo-Goodwinian features, drawing heavily on Barbosa-Filho and Taylor (2006).[5] In this seminal piece, the authors postulate *ad hoc* growth functions for output, capacity, real wages and labor productivity, all of which take the labor share and the rate of utilization as arguments. Here, we utilize the same model structure, but assume that capacity growth is defined by the warranted rate of growth. Further, we simplify real wage and productivity functions to zoom in on the relevant causal connections.

First, we present the model with profit-led activity and profit-squeeze distribution. Asymptotic stability and cyclical convergence in a Goodwin pattern follow therefrom. We then derive comparative dynamic results for an adverse shock to labor market institutions, which is introduced via a parameter $\alpha$ in the real wage Phillips curve; i.e. $\alpha \downarrow$. Key results are that the warranted rate of growth is predicted to increase, whereas the natural rate of growth is predicted to decrease. The change in the warranted rate of growth is driven by an increase in the income-capital ratio, which dominates the decrease in the labor share. In conclusion, neo-Goodwinian growth without a Harrodian labor constraint implies a divergence between warranted and natural growth, and also does not conform to the stylized facts of neoliberalism.

*A model in $u$ and $\psi$*

The distributive cycle model of this section features two laws of motions:

$$\dot{u} = u(h - g^w) \tag{3.1}$$

$$\dot{\psi} = \psi(\omega - a), \tag{3.2}$$

where $h = \hat{Y}$ is the independent expenditure function that determines output growth. Income-capital ratio and labor share are the state variables; all terms have been previously defined.

These laws of motions are brought to life with the warranted rate of growth $g^w$, and three behavioral functions to describe the growth of output $h$, endogenous labor productivity growth $a$, and real wage Phillips curve $\omega$:

$$g^w = s_\pi(1 - \psi)u \tag{3.3}$$

[5] See Barrales et al. (2021b, Section 2) for an overview of theoretical motivations and further references.

$$h = h(u, e, \psi), h_u > 0, h_e < 0, h_\psi < 0 \tag{3.4}$$

$$a = a(\psi), a_\psi > 0 \tag{3.5}$$

$$\omega = \omega(u), \omega_u > 0. \tag{3.6}$$

The partials can be motivated as follows: First, $h_u$ is positive, as in Skott (1989): a higher level of demand as proxied by a higher income-capital ratio $u$ leads to an increase in the growth rate of output. Barbosa-Filho and Taylor (2006) assume $h_u < 0$, which immediately satisfies dynamic stability of the activity variable. However, in Skott's framework and also the three-dimensional model in Section 4, the inclusion of the employment rate requires the assumption that increases in demand lead to increases in the employment rate.

The employment rate is exogenous in the model of this section—but we can already introduce the relevant partial, $h_e < 0$. Skott (1989, p. 236) motivates this sign as a decrease in the desired rate of expansion due to adjustment and turnover costs at high employment rates. The sign can also be motivated with direct reference to Kalecki's seminal essay on the "political aspects of full employment:" high employment rates undermine the power of capital, and thus depress expansion plans. Last but not least, $h_\psi < 0$ represents a Kaleckian link from functional distribution of income to economic activity, although here driving investment as expenditure first.[6] In summary, $h$ is an independent expenditure function that renders the model Keynesian-Kaleckian.

The induced technical change effect ($a_\psi > 0$) implies that higher real wages relative to labor productivity trigger efforts to economize on this production factor's costs. It can be motivated in optimizing frameworks (Kennedy, 1964; Shah and Desai, 1981; Foley et al., 2019). Here we model aggregate productivity directly, and simply assume the pressure for labor-saving innovation when real unit labor costs are high (Barbosa-Filho and Taylor, 2006; Storm and Naastepad, 2012). Lastly, in the model of this section, the profit squeeze works through the activity variable $u$, as the labor market is relegated to the background. This is as in other Goodwin-Kalecki models and related empirical applications.[7]

The nonlinear two-dimensional system of differential equations is given by substitution

[6] Specifications vary across the literature. Commonly, the time rate of change of the rate of utilization (or the income-capital ratio) is specified to close a gap between accumulation $g$ and capacity savings $g^w$, where $g$ takes a Kalecki-Steindl or Bhaduri-Marglin form. In other approaches, and ours here, the growth rate of *output* is presumed to trade off against capacity. Hence, $h_\psi < 0$ already assumes profit-led activity. However, $h_\psi + s_\pi \pi$ determines the slope of the nullcline for the income-capital ratio, and we therefore focus on this latter inequality. See also the discussion just after equation 3.7 below.

[7] Again, see Barbosa-Filho and Taylor (2006). This "profit squeeze" can be motivated on the grounds that Keynesian (un)employment is determined primarily by the level of output, so that an increase in $u$ implies an incease in $e$, which then drives real wage increases. Alternatively, employed "insiders" drive wage increases on the basis of activity levels within the plant or firm, leaving the unemployed outsiders little sway.

of equations 3.3–3.6 into the two laws of motion. The Jacobian matrix at the non-trivial steady state follows as:

$$J^* = \begin{bmatrix} u(h_u - s_\pi \pi) & u(h_\psi + s_\pi u) \\ \psi \omega_u & -\psi a_\psi \end{bmatrix}, \tag{3.7}$$

where we refrain from starring variables for brevity. We assume $h_u < s_\pi \pi$ (i.e., own-stability of $u$) and $|h_\psi| > s_\pi u$ (i.e. profit-led demand), and with these assumptions the sign pattern is unambiguous.[8] $Tr(J^*) < 0, |J^*| > 0$ guarantee asymptotic stability: the model converges in a stable focus to the non-trivial steady state. The dampened cycle is of Goodwin type, which implies that the income-capital ratio leads the labor share. Equivalently, the direction of the stable focus is counter-clockwise in $u, \psi$-plane.

*Labor suppression*

Now consider a change in labor market institutions that weakens the bargaining power of labor. Bargaining is formalized in the model by the real wage Phillips curve, which in the model of this section responds to the income-capital ratio. We assume a shift parameter $\alpha$, so that equation 3.6 becomes $\omega(u; \alpha)$ with $\omega_\alpha > 0$.

Key results regarding state variables are:

$$\frac{\partial u^*}{\partial \alpha} = \frac{\begin{vmatrix} 0 & u(h_\psi - s_\pi u) \\ -\psi \omega_\alpha & -\psi a_\psi \end{vmatrix}}{|J^*|} < 0 \tag{3.8}$$

$$\frac{\partial \psi^*}{\partial \alpha} = \frac{\begin{vmatrix} u(h_u - s_\pi \pi) & 0 \\ \psi \omega_u & -\psi_\alpha \end{vmatrix}}{|J^*|} > 0 \tag{3.9}$$

and it follows therefrom that

$$\frac{\partial g^w}{\partial \alpha} = s_\pi (1 - \psi) \frac{\partial u}{\partial \alpha} - s_\pi u \frac{\partial \psi}{\partial \alpha} < 0 \tag{3.10}$$

$$\frac{\partial g^*}{\partial \alpha} = a_\psi \frac{\partial \psi}{\partial \alpha} > 0. \tag{3.11}$$

The first result to emphasize is that the advsere shock to labor market institutions indeed affects the labor share. This is not the case in models with an innovation

[8] We will not relitigate the question of demand regimes: no convincing empirical evidence for short run wage led demand exists. See Barrales et al. (2021b) and references therein. Relevant research questions regarding the Goodwin pattern exist; these concern the weakening of the profit squeeze and the role of pro-cyclical labor productivity over the course of the cycle (Setterfield, 2021), but not the Goodwin pattern itself.

possibility frontier, unless additional assumptions are made.[9] As a consequence, the steady state labor share falls if labor is weakened.

Second, the income-capital ratio changes in the opposite direction as $\alpha$. Labor suppression unambiguously leads to an increase in $u$. The reason is that labor suppression unambiguously expands output growth, *barring other limits on $h$.* The standard phase diagram with a positively sloped profit squeeze isocline and a negatively sloped goods market isocline also illustrates this: the downward shift in $\dot{\psi} = 0$ implies a higher $u^*$.

The increase in $u$ is possible since $g^w \neq g^*$. No mechanism exists to equilibrate warranted and natural rates of growth, and the former can rise even though the latter declines. The next section presents a model that enforces a solution to Harrod's first problem.

# 4 Adding the labor constraint: Balanced growth and stagnation

We now add the employment rate as state variable, and shift the burden of the profit squeeze squarely into the labor market. The employment rate's law of motion requires equalization of output growth and natural growth in steady state. As in the previous section, the law of motion for the income-capital ratio requires equalization of output growth and warranted growth in steady state. Put together, the three-dimensional model solves Harrod's first problem.

An adverse shock to labor's bargaining power implies a fall in the labor share, a fall in the income-capital ratio and a fall of steady state growth. An adverse (public) demand shock generates the same signs for these variables, but pushes the employment rate in the opposite direction as the bargaining shock. In other words, and in combination, these shocks lead to high inequality and low growth, while the employment rate could change either way. Thus, the model's key variables match the stylized facts of neoliberalism.

*A model in $u, e$ and $\psi$*

The model of this section features three laws of motion. Specifically, the model adds the employment rate $e = L/N = Y/(AN)$ as state variable. The three-dimensional system of nonlinear differential equations is

$$\dot{u} = u(h - g^w) \tag{3.1}$$

[9] In short, the curvature of the innovation possibility frontier fully determines the labor share of income as the outcome of the firm's optimal technological choice. Hence, as in the classical growth cycle of Goodwin (1967), the functional distribution of income is invariant to parameters of the Phillips curve. See also the discussion near the end of Chapter 7 in Foley et al. (2019), Petach and Tavani (2020) and Rada et al. (2021).

$$\dot{e} = e(h - (a + n)) \tag{4.1}$$

$$\dot{\psi} = \psi(\omega - a). \tag{3.2}$$

The law of motion for the employmnent rate defines the time rate of change of $e$ as a function of the difference between realized growth rates of output $\hat{Y} = h$ and effective labor force $g^* = a + n$. The laws of motion for income-capital ratio and labor share are as in the previous section.

The model includes equations 3.3–3.5, but the real wage Phillips curve now responds directly to the employment rate:

$$\omega = \omega(e), \omega_e > 0. \tag{4.2}$$

Substituting these four equations into the three laws of motion gives the following Jacobian matrix, evaluated at the non-trivial steady state:

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) \\ eh_u & eh_e & e(h_\psi - a_\psi) \\ 0 & \psi\omega_e & -\psi a_\psi. \end{bmatrix} \tag{4.3}$$

We assume $h_u < s_\pi(1-\psi)$ to obtain stable own-feedback of the income-capital ratio, and $|h_\psi| > s_\pi u$ to obtain profit-led demand (see also footnote 8). The sign pattern follows as

$$J^* = \begin{bmatrix} - & - & - \\ + & - & - \\ 0 & + & - \end{bmatrix}. \tag{4.4}$$

An appendix in Rada et al. (2021) lists the Routh-Hurwitz conditions and discusses stability, which is satisfied under plausible restrictions.[10]

Further, the model generates relevant cyclical stylized facts (Zipperer and Skott, 2011; von Arnim and Barrales, 2015; Barrales et al., 2021b). The two-dimensional subsystems are consistent with real world cycles in $u, e$ and $e, \psi$. The $u, \psi$-cycle emerges only in the three-dimensional system, and is there determined by $\partial\dot{e}/\partial u = eh_u > 0$:

[10] The first three Routh-Hurwitz inequalities are always satisfied, given the assumptions on signs in the Jacobian 4.4. A sufficient condition for the fourth inequality to hold is $-(eh_e - \psi a_\psi) > uh_u$. While this is only sufficient, it is straightforwardly interpreted: the stabilizing elements along the trace have to outweigh the destabilizing element $h_u$. In particular, $h_u$ appears in the law of motion of the employment rate, and there can lead to violation of the fourth Routh-Hurwitz inequality. Note further that the fourth Ruth-Hurwitz inequality ensures that the real parts of a potential pair of complex eigenvalues are negative. In numerical simulations it was confirmed that increases in $h_u$ lead to a Hopf bifurcation as the real parts pass through zero from below. A stable limit cycle emerges (with linear behavioral functions). The assumption of asymptotic stability facilitates comparative dynamic exercises, which are our focus here. This result, however, suggests that the model could plausible generate endogenous fluctuations, which is appealing for a theory of cyclical growth. Details are available upon request. See Barrales and von Arnim (2021) for a related investigation.

the employment rate increases in the income-capital ratio, and then drives the profit squeeze via $\omega_e$.

*Labor suppression, and other comparative dynamics*

The three-dimensional model features principally the same causal linkages as the simpler model of the previous section. The crucial difference is that the growth rate of output is forced to equilibrate with the warranted rate of growth (in equation 3.1) and additionally with the natural rate of growth (in equation 4.1). This of course implies also that $g^w = g^*$. As before, the labor share is tied to institutions governing real wage bargaining as described by $\alpha$, so that

$$\frac{\partial g^*}{\partial \alpha} = a_\psi \frac{\partial \psi}{\partial \alpha} > 0 \Leftrightarrow \frac{\partial g^w}{\partial \alpha} > 0. \tag{4.5}$$

Table 2 summarizes these and related results. Appendix A.1 provides details. We assume that the direct impact of changes in $n$ and $a$ on the steady state rate of growth dominates the indirect impact via the labor share. See equations A.12 and A.16. We emphasize three main points.

First, the sign patterns of the first two columns determine this model to be Keynesian. The paradox of thrift holds in the long run: a change in the savings propensity or a shift in the expenditure function have steady state effects on rates of utilization, employment and growth. Importantly, an adverse shock to the expenditure function ($h_0 \downarrow$) triggers declines in all three state variables and the steady state growth rate. As discussed in Section 2, such a shock could describe the withdrawal of public support across a range of expenditure categories, from transportation and energy infrastructure to R&D and public education.

Second, shocks to supply side factors do not produce steady state changes in line with observed trends of stagnation. A decrease in labor supply growth $n$ or a slowdown in innovative potential $a_0$ leads to a decrease in the steady state growth rate, *but also places upward pressure on the labor share.* To be sure, this does not mean that trend changes à la Gordon (2016) play no role in stagnationary tendencies. However, situating these as *causae causans* requires a separate explanation for the decline of the labor share. In neoclassical approaches, the elasticity of substitution then takes center stage (see Piketty, 2013, among others).

Third, the signs across the last column, summarizing results for a bargaining shock, conform to the stylized facts of neoliberalism. A decrease in $\alpha$—conceived of as a shift in the intercept of the real wage Phillips curve in equation 4.2 or a decline in its slope parameter near the steady state—leads to declines in $u^*$, $\psi^*$, $g^w$ and $g^*$. Specifically, in the warranted rate of growth the decline in the income-capital ratio dominates the rise of the profit share—thus allowing for convergence of $g^w = g^*$.

In summary, the causal linkages of labor-constrained growth imply, as in the classical growth cycle of Goodwin (1967), that adverse shocks to $a$ and $n$ favor labor. However, in the Keynesian structuralist version laid out here, the labor share is linked in steady state to institutions of the labor market, rather than only technology. In this view, the neoliberal state, asked to retreat in the face of excessive faith in markets, and the neoliberal labor market, deregulated and deskilled to favor capital, join forces to depress labor share, income-capital ratio and steady state growth, but generate countervailing forces on the employment rate.

## 5 The utilization controversy in the current context

In the three-dimensional model of the previous section the income-capital ratio is endogenous to demand in the long run, the paradox of thrift holds in the long run, and, through variations in $u$, the warranted rate of growth is endogenous to demand in the long run. Critiques of these features have spawned a literature on the "utilization controversy." We will not review this literature in detail, but merely provide some background to motivate the model of this section. Our main objective is to illustrate that the key conclusion of the model of Section 4—labor suppression causes a decline in labor share and steady state growth rate—holds also in an extended model where the rate of utilization converges to an exogenously given desired rate in the long run.

To fix ideas, recall that $u = \sigma U = (Y^*/K)(Y/Y^*)$ by definition. Now suppose that there exists a *desired* rate of utilization $\bar{U}$. Therefore, given the technical coefficient, there exists a $\bar{u} = \sigma\bar{U}$. Given the savings rate $s = s_\pi(1-\psi)$, the steady state rate of growth is fully determined, and constant as long as the desired rate of utilization or the technical coefficient do not change: $g^w = s\sigma\bar{U}$. Put differently, to preserve a role for demand in the long run, either the savings rate or the desired rate of utilization must become endogenous.[11]

Sraffian supermultiplier theory renders the (ex post) savings rate endogenous, via the *investment rate*. Importantly, the theory makes a clean distinction between the long run endogeneity of growth and the short run paradox of thrift. In particular, the warranted rate of growth is endogenous to demand, which is driven by the exogenous growth rate of autonomous expenditures. In contrast, the savings *propensity* has an effect on economic activity only in the short run (Freitas and Serrano, 2015; Nikiforos et al., 2021). The steady state rate of utilization is exogenous and constant, and the warranted rate depends on the *investment share* in output.

Nikiforos (2013) instead proposes that the desired rate of utilization becomes en-

[11] In models with a micro-founded innovation possibility frontier, the income-capital ratio—here a purely *technological* variable—also becomes endogenous; see references above on Kennedy-style frameworks. Other recent research motivates an endogenous rate of utilization with game-theoretic interactions between firms; see Petach and Tavani (2021). These are all plausible ideas to pursue, but our focus here lies on Keynesian theoretical traditions.

dogenous. Specifically, $\bar{U}$ is the result of cost-minimization, which in turn depends on scale effects. In consequence, the *optimal* rate of utilization is a function of demand, also in the long run. (This is a substantially different argument than evolving norms, as proposed in "zero root" models.) However, in this framework, a decline in the savings propensity would also lead to a decline in the desired rate of utilization and warranted rate of growth. Many classical authors see such an application of the Keynesian paradox of thrift to the long run as questionable at best.

Nikiforos (2020) provides a summary of this debate, and in a companion piece also discusses empirics that illustrate how various measures of capital utilization show significant medium-term variation throughout the post-war era. The crux of the matter is that these things are hard to measure. As discussed in Section 2, proxies for $u$ rely on estimates of "capacity" that are often limited. The technical coefficient $\sigma$, the rate of utilization $U$ and its desired level $\bar{U}$ are all unobservable. If observed $u$ has declined, it could be due to a fall in $\sigma$ that dominates a rise in $U$, or vice-versa. Whether observed $u$ is at or near its steady state level at any point in time is impossible to assess without making strong theoretical assumptions.

In summary, observed $u$ appears to have decreased during the neoliberal era, but we cannot claim with confidence why that might be so. However, an exogenously given and constant $\bar{u} = \sigma\bar{U}$ has enduring appeal for some authors in the classical-Keynesian traditions. Hence, we present now a four-dimensional model where the steady state rate of utilization $u^* = \bar{u}$.

Specifically, we endogenize the savings propensity of capitalists. Its time rate of change increases when $u > \bar{u}$, and vice versa. The simple structure is essential: the closure variable can respond only to the difference between realized and desired income-capital ratio—which features the same dynamics as the realized and desired rate of utilization, assuming the technological full capacity output to capital ratio to be constant. Blecker and Setterfield (2019, p. 288) discuss such a formalization, with reference to Shaikh (2009), who in turn draws on Kalecki's insight that capacity savings are undertaken by firms rather than households.

The motivation for this particular choice thus requires interpretation of $s_\pi$ as firm's retained earnings. A more complete accounting would encompass firms directly. For an overview of these accounting relationships see Foley et al. (2019, Ch. 14), but in the interest of space we will not pursue that route here. In a nutshell, the idea of the endogenous savings propensity is that expectations of boom conditions spark accumulation and in its wake an increase in firm's retention rate. The latter rises to finance (a portion of) the former. In consequence, the steady state growth rate changes while the rate of capacity utilization returns to the same desired level.

Hence, the traverse is Keynesian but the steady state classical.[12] The paradox of

[12] Duménil and Lévy (1999) carries this distinction in the title. Their approach uses *prices* as the

thrift matters only in the short run, while the long run converges to an exogenously given and constant desired rate of utilization. All of this finds expression in the model presented just below. Labor suppression triggers a decline in the labor share which stimulates short run accumulation and income-capital ratio. In contrast to the model of the previous section, the savings propensity follows. At the same time, the natural rate of growth declines. In steady state, the income-capital ratio has returned to its initial value, and the burden of adjustment in the warranted rate of growth falls on the savings propensity.

Michl (2017) suggests a narrative that can be usefully juxtaposed. The author proposes that increases in profitability will not lead to a boom in accumulation when the propensity to invest is weak. Instead, an "unproductive" feedback loop between stock market valuations and inequality is triggered. Here, we do not account for financial factors, but the boost to profitability from labor suppression dissipates in steady state into a falling "real" savings propensity. This implies a reduction in capacity investment, and hence stagnation.

*A model with an endogenous propensity to save*

The model of this section is four-dimensional. The three laws of motion are restated from above, and the fourth equation adds the savings propensity of capitalists as endogenous:

$$\dot{u} = u(h - g^w) \tag{3.1}$$
$$\dot{e} = e(h - (a + n)) \tag{4.1}$$
$$\dot{\psi} = \psi(\omega - a). \tag{3.2}$$
$$\dot{s}_\pi = \beta(u - \bar{u}), \tag{5.1}$$

where $\beta > 0$ is a speed of adjustment coefficient. In a nutshell, the savings propensity rises when the income-capital ratio is above its desired level $\bar{u}$, and vice versa. $h, g^w, a$ and $\omega$ are as before in equations 3.3–3.5 and the real wage Phillips curve in $e$, equation 4.2.

The Jacobian matrix of this system, evaluated at the non-trivial steady-state, is:

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) & -u(1-\psi) \\ eh_u & eh_e & e(h_\psi - a_\psi) & 0 \\ 0 & \psi\omega_e & -\psi a_\psi & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}, \tag{5.2}$$

closure variable to achieve $u^* = \bar{u}$. Such closures can be incompatible with a profit squeeze, and can imply a cycle that is contradicted by the empirically observed Goodwin pattern. See von Arnim and Barrales (2015) for a critical discussion.

and, assuming as previously own-stability of $u$ and profit-led demand, its sign pattern follows as:

$$J^* = \begin{bmatrix} - & - & - & - \\ + & - & - & 0 \\ 0 & + & - & 0 \\ + & 0 & 0 & 0 \end{bmatrix}. \tag{5.3}$$

It is clear from inspection that the trace is negative. Appendix A.2 shows that the determinant is positive. Both of these are necessary though not sufficient conditions for all four roots to have negative real parts. The four-dimensional Routh-Hurwitz conditions might provide a route forward, but in the interest of space we do not pursue a proof. This might be justifiable in light of the fact that two important necessary conditions are satisfied. Moreoever, the savings propensity interacts only with the income-capital ratio and serves there as an automatic stabilizer: higher demand triggers an increase in the savings propensity and hence a decline in the multiplier. Since the three-dimensional model is stable, the addition of an automatic stabilizer will not change everything.[13] Last but not least, the model does just like that of Section 4 generate relevant stylized facts of cyclical growth.

*Labor suppression, and a demand shock*

The purpose of this last set of comparative dynamics is to highlight that the four-dimensional model exhibits a fall in the labor share and secular stagnation in response to labor suppression—like the three-dimensional setup of the previous section. However, unlike that model, we here also obtain convergence of $u$ to a constant $\bar{u}$ in the long run. As a result, the paradox of thrift becomes inoperable in the long run. In any new steady state, investment and savings share are equal—but an expansionary demand shock triggers a multiplier effect, and an increase in the savings propensity itself. This latter effect persists until $u = \bar{u}$. Nevertheless, the steady state rate of growth is endogenous to demand changes, since the natural rate responds to the labor share ($\partial g^*/\partial\psi > 0$), and the warranted rate responds to the labor share ($\partial g^w/\partial\psi < 0$) *and* additionally the savings rate ($\partial g^w/\partial s_\pi > 0$).

We will not lay out all possible scenarios. Of course, the income-capital ratio is now invariant to parameter changes. The employment rate still serves as the activity variable that mediates class conflict: it responds inversely to changes in institutions that govern the distribution of income. In what follows, we focus on steady state labor share and growth rate. Specifically, we consider the labor suppression shock (i.e. $\alpha$) and a demand shock (i.e. $h_0$). Since the warranted growth rate converges to the natural rate through changes in $s_\pi$, it suffices to describe $\psi^*$ and how $g^*(\psi^*)$ responds to it.

[13]Further, numerical simulations indicate that the four-dimensional model is indeed stable over reasonable parameter ranges. Details are available upon request.

Key results are

$$\frac{\partial\psi^*}{\partial\alpha} = \frac{|J^{\psi,\alpha}|}{|J^*|} > 0 \Rightarrow \frac{\partial g^*}{\partial\alpha} = a_\psi \frac{\partial\psi}{\partial\alpha} > 0 \tag{5.4}$$

$$\frac{\partial\psi^*}{\partial h_0} = \frac{|J^{\psi,h_0}|}{|J^*|} > 0 \Rightarrow \frac{\partial g^*}{\partial h_0} = a_\psi \frac{\partial\psi}{\partial h_0} > 0. \tag{5.5}$$

Appendix A.2 provides details. In short, (i) labor suppression as proxied by the parameter $\alpha$ in the real wage Phillips curve leads to a fall in the labor share, and subsequently depresses the natural rate of growth through the channel of induced technical change; and (ii) "austerity" as proxied by the parameter $h_0$ in the independent expenditure function leads to an initial fall in the income-capital ratio, and in the resulting steady state, labor share and growth rate are permanently lower.

## 6 Conclusions

A number of classical and Keynesian contributions have addressed the pressing contemporary issue of high inequality and poor macroeconomic performance. A dominant strain of thought still appears to be that insufficient demand—and, in particular, insufficient consumption demand due to high inequality—constrains growth in the long run (Blecker, 2016; Hein, 2016; Storm, 2018). Models constructed around this theme do not fit *cyclical* stylized facts well. In contrast, classically inspired research puts the mechanism of induced technical change at the heart of the story (Petach and Tavani, 2020), and some authors have augmented this with costly (potentially public) R&D efforts (for an overview, see Tavani and Zamparelli, 2017, Section 6). These latter contributions make clear distinctions between short and long run, and give *institutions* that drive policy a prominent role. However, realized and warranted rate of growth are equal by assumption: not even the short run features aggregate demand problems, and in the long run the savings propensity picks up double duty to describe investment behavior.[14]

Our intention here is to meet both approaches halfway. Theory presented in this paper is Keynesian-Kaleckian in the sense that demand drives growth. Additionally, the distribution of income is linked to labor market institutions, not only technology. However, growth is also labor constrained. A classical trade-off between employment rate and labor market institutions is implicit. Key results show that labor-constrained versions of the model *even with forced convergence to a desired rate of utilization* generate low growth and high inequality as a response to labor suppression. Public expenditure reductions produce analogous outcomes.

[14] Other candidate causes might matter greatly: structural change, globalization and financialization all plausibly play a role. However, these are not the focus of the investigation here. On the issue of structural in particular, see Mendieta-Muñoz et al. (2020); Barrales et al. (2021a); Rada et al. (2021).

Labor suppression is proxied by an adverse shock to the real wage Phillips curve, and "austerity" by an adverse shock to the independent expenditure function. Both of these are imperfect. Models are abstractions, as are the thought experiments imposed upon them. Still, the gist of our argument appears to hold: capital's successful efforts to impede the societal provision of infrastructure and institutions have critically undermined the sustainability of the macroeconomy. The key conclusion is that stagnation and inequality are the result of social conflict and bad policy, not inexorable demographic and technological trends.

Last but not least, while Goodwinian theory is rooted in Marxian thought, it outlines how capitalism works—at best. In this sense, problems and prescriptions weighed here are boilerplate social democratic. The task is to reduce conflict and overcome coordination failures, and doing so could at least in theory overcome both demand and supply side causes of high inequality and stagnation.

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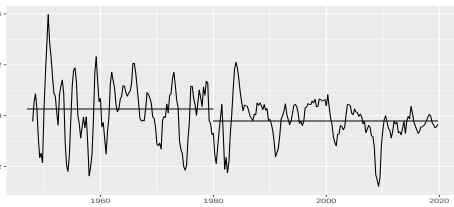
## 8 Figures & Tables

| | Fig. 1 | Source | Pre-1980 | Post-1980 |
|---|---|---|---|---|
| Real GDP (%Δ) | (a) | GDPC1 | 0.039 | 0.026 |
| Income-capital ratio | (b) | (see caption) | 0.675 | 0.622 |
| Labor Share | (c) | PRS85006173 | 112.5 | 106.7 |
| Employment rate (solid) | (d) | 1-UNRATE | 94.8 | 93.8 |
| Employment rate (25-54yo) | (d) | LNS12300060 | 67.5 | 78.2 |
| Civilian Labor Force (%Δ) | (e) | CLF16OV | 0.018 | 0.011 |
| Output per Hour (%Δ) | (f) | OPHPBS | 0.029 | 0.019 |
| Real Wage (%Δ) | (g) | COMPRNFB | 0.022 | 0.010 |
| Public expenditures (% of GDP) | (h) | (see caption) | 0.131 | 0.085 |

**Table 1: Stylized facts, pre- and post-1980**. The table lists period averages (pre-1980 and post-1980) for the series displayed in standardized form in Figure 1. First column lists panels of that figure. All series except the income-capital ratio are downloaded from the Federal Reserve Economic Database (FRED). The second column lists FRED series codes. The income-capital ratio is constructed as the ratio of corporate value added to current-cost fixed assets from NIPA Tables 1.14 and 6.1. The last row reports the ratio of key federal expenditures relative to GDP. Specifically, the numerator is the sum of FRED series A787RC1Q027SBEA, Y069RC1Q027SBEA and A957RC1Q027SBEA, which measure federal gross investment, federal non-defense R&D measures and federal consumption, respectively. %Δ denotes percent change compared to quarter one year ago.

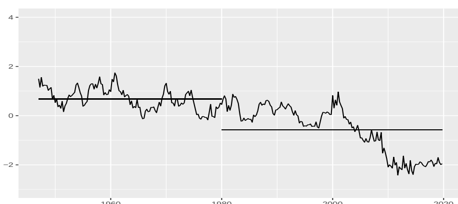
| | $s_\pi$ | $h_0$ | $n$ | $a_0$ | $\alpha$ |
|---|---|---|---|---|---|
| Income-capital ratio $u^*$ | $-$ | $+$ | $+$ | $+$ | $+$ |
| Employment rate $e^*$ | $-$ | $+$ | $-$ | $+$ | $-$ |
| Labor share $\psi^*$ | $-$ | $+$ | $-$ | $-$ | $+$ |
| Growth rate $g^* = g^w$ | $-$ | $+$ | $+^*$ | $+^*$ | $+$ |

**Table 2: Comparative dynamics**. The table summarizes results of comparative dynamic exercises for the model of Section 4. Details are provided in Appendix A.1. Starred signs ($\partial g^*/\partial n$ and $\partial g^*/\partial a_0$) assume that the direct impact of the parameter on the natural growth rate dominates the indirect impact via the labor share.

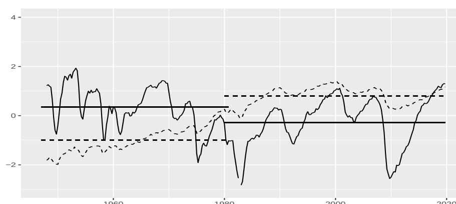



**(a)** Growth rate of real GDP $h$

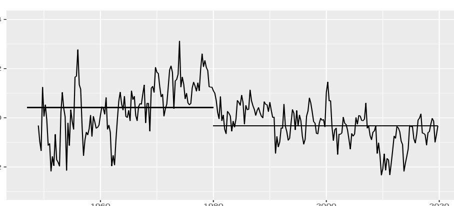

**(b)** Income-capital ratio $u$



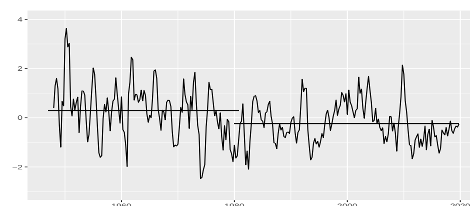

**(c)** BLS “headline” labor share $\psi$



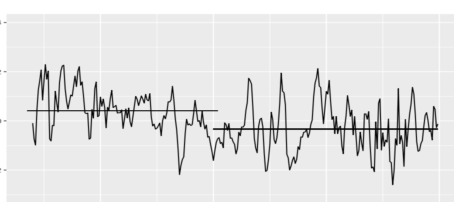

**(d)** Employment rate $e$



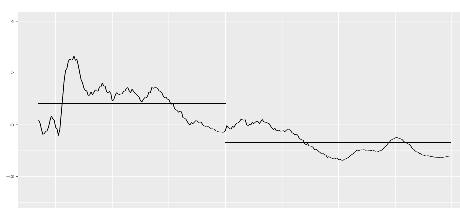

**(e)** Growth rate of labor force $n$




**(f)** Growth rate of labor productivity $a$




**(g)** Growth rate of real wage $\omega$



**(h)** Key public expenditures

**Figure 1: Stylized facts**. Panels (a)–(h) show key time series in standardized form. Horizontal lines are standardized series averages for pre-1980 and post-1980 periods. See Table 1 for sources and raw series averages, and Section 2 for discussion.

# A Appendices

## A.1 Appendix to Section 4

This appendix presents detail on the comparative dynamics summarized in Table 2. We consider changes in savings rate $s_\pi$ and labor force growth rate $n$, which are both exogenous parameters. Further, we assume shift terms in the growth function (i.e., $h_0$), productivity function ($a_0$), and the here centrally important bargaining shock in the real wage Phillips curve ($\alpha$). We utilize Cramer's rule. For brevity, we label $\lambda = 1/|J^*| < 0$, with $|J^*|$ given in Equation 4.3.

**Savings propensity** $s_\pi$

$$\partial u^*/\partial s_\pi = \lambda(1-\psi)u^2|J_{11}| < 0 \tag{A.1}$$
$$\partial e^*/\partial s_\pi = -\lambda(1-\psi)u^2|J_{12}| < 0 \tag{A.2}$$
$$\partial \psi^*/\partial s_\pi = \lambda(1-\psi)u^2|J_{13}| < 0 \tag{A.3}$$
$$\partial g^*/\partial s_\pi = a_\psi \partial\psi/\partial s_\pi < 0 \tag{A.4}$$

**Demand** $h_0$

$$\partial u^*/\partial h_0 = \lambda\left(-u|J_{11}| + e|J_{21}|\right) = -\lambda ue\psi(a_\psi + s_\pi u)\omega_e > 0 \tag{A.5}$$
$$\partial e^*/\partial h_0 = \lambda\left(u|J_{12}| - e|J_{22}|\right) > 0 \tag{A.6}$$
$$\partial \psi^*/\partial h_0 = \lambda\left(-u|J_{13}| + e|J_{23}|\right) > 0 \tag{A.7}$$
$$\partial g^*/\partial h_0 = a_\psi \partial\psi/\partial h_0 > 0 \tag{A.8}$$

**Demographics** $n$

$$\partial u^*/\partial n = -\lambda e|J_{21}| > 0 \tag{A.9}$$
$$\partial e^*/\partial n = \lambda e|J_{22}| < 0 \tag{A.10}$$
$$\partial \psi^*/\partial n = -\lambda e|J_{23}| < 0 \tag{A.11}$$
$$\partial g^*/\partial n = 1 + a_\psi\partial\psi/\partial n > 0 \Leftrightarrow |a_\psi\partial\psi/\partial n| < 1 \tag{A.12}$$

**Productivity** $a_0$

$$\begin{aligned}\partial u^*/\partial a_0 &= \lambda\left(-e|J_{21}| + \psi|J_{31}|\right) \\ &= \lambda ue\psi(\omega_e(h_\psi + s_\pi u) + h_e s_\pi u) > 0\end{aligned} \tag{A.13}$$
$$\partial e^*/\partial a_0 = \lambda\left(e|J_{22}| - \psi|J_{32}|\right) = \lambda ue\psi(s_\pi \pi h_\psi + s_\pi u h_u) > 0 \tag{A.14}$$
$$\partial \psi^*/\partial a_0 = \lambda\left(-e|J_{23}| + \psi|J_{33}|\right) < 0 \tag{A.15}$$
$$\partial g^*/\partial a_0 = 1 + a_\psi\partial\psi/\partial a_0 > 0 \Leftrightarrow |a_\psi\partial\psi/\partial a_0| < 1 \tag{A.16}$$

To show that $\partial e^*/\partial a_0 > 0$ despite $h_\psi < 0$ after the second equal sign, (i) multiply $j_{11} < 0$ by $u$ and $j_{13} < 0$ by $\pi$, and (ii) add these two see that $\pi h_\psi + u h_u < 0$. Assuming stability in $u$ and profit-led demand requires a sufficiently "small" $h_u$, and in consequence this positive employment effect. See also footnote 10.

**Bargaining $\alpha$**

$$\begin{aligned}\partial u^*/\partial\alpha &= -\lambda\psi|J_{31}| = \lambda ue\psi(a_\psi + s_\pi u)h_e > 0 & \text{(A.17)}\\ \partial e^*/\partial\alpha &= \lambda\psi|J_{32}| < 0 & \text{(A.18)}\\ \partial\psi^*/\partial\alpha &= -\lambda\psi|J_{33}| > 0 & \text{(A.19)}\\ \partial g^*/\partial\alpha &= a_\psi\partial\psi/\partial\alpha > 0 & \text{(A.20)}\end{aligned}$$

## A.2 Appendix to Section 5

First, we show that $|J^*| > 0$. Expanding along the fourth column gives

$$|J| = -j_{14}|J_{14}| = -j_{14}\left(j_{41}|M|\right) > 0, \tag{A.21}$$

since

$$-j_{14} = u(1-\psi) > 0 \tag{A.22}$$

$$j_{41} = \beta > 0 \tag{A.23}$$

$$|M| = \begin{vmatrix} eh_e & e(h_\psi - a_\psi) \\ \psi\omega_e & -\psi a_\psi \end{vmatrix} = \begin{vmatrix} - & - \\ + & - \end{vmatrix} > 0. \tag{A.24}$$

Comparative dynamic results require to sign two further determinants:

$$|J^{\psi,\alpha}| = -j_{14}\left|J_{14}^{\psi,\alpha}\right| = u(1-\psi)\psi a_\psi(-eh_e\beta) > 0 \tag{A.25}$$

$$|J^{\psi,h_0}| = -j_{14}\left|J_{14}^{\psi,h_0}\right| = u(1-\psi)(-e)(-\psi\omega_e\beta) > 0 \tag{A.26}$$