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Abstract

We examine the equilibrium wage and employment outcomes in a labor market model comprised of informationally constrained workers and employers whose labor market interactions have a non-zero impact on wages. The model endogenizes employment interactions between workers and employers in terms of a quantal response equilibrium and produces an equilibrium level of frictional unemployment as a statistical feature of a decentralized labor market. Shocks to the economy can produce short-run equilibrium involuntary unemployment arising from unfulfilled expectations. Even after agents align their expectations with market outcomes, unless they also adjust their expectations of the scale of statistical fluctuations in wages, a negative shock to demand can result in higher levels of equilibrium unemployment. In this way the model exhibits a particular type of non-neutrality of money in the short-run and long-run.

Keywords: Unemployment, Unfulfilled expectations, Wage distribution, Labor market, Statistical equilibrium.
JEL Classification: C18, D80, E10, E24, E70

1 Introduction

Labor market interactions between workers and employers determine aggregate employment and wage outcomes that can be understood in terns of equilibrium frequency distributions [Foley, 1994, 1996]. When workers and employers face information-processing constraints their actions are described probabilistically by logit-quantal response distributions, which are defined by agents' payoffs and expectations. Quantal responses in actions induce strong correlations between wages and employment interactions that lead to a non-degenerate wage distribution and persistent unemployment in equilibrium. When workers' and employers' hiring interactions primarily depend on the wage the labor market can be represented by a joint distribution over each agent's actions and the wage level. Equilibrium is the joint distribution that maximizes informational entropy subject to the behavioral and institutional constraints of competitive labor market interactions.

In equilibrium agents' expectations about the wage are fulfilled and endogenous fluctuations in the frequency distribution of wages produce persistent frictional unemployment as a consequence of decentralized labor market interactions and agents' information-processing constraints. Exogenous shocks to the statistical equilibrium produce involuntary unemployment due to unfulfilled expectations. When shocks are permanent, agents' expectations can adjust to a new statistical equilibrium at levels of unemployment above or below those prior to the shock.

One important implication of our model is that changes in a nominal exogenous variable due, for example, to a change in monetary policy or aggregate demand, can result in the uneven adjustment of agents' expectations leading to real changes in the wage and level of unemployment. While Walrasian equilibrium level of unemployment is the level that remains after the "grinding out" of such changes to the broader economic environment, statistical equilibrium unemployment "builds in" these changes. In these situations money is no longer necessarily neutral in the long-run.

2 Entropy Constrained Behavior

Following Scharfenaker and Foley [2017]; Scharfenaker [2020]; Foley [2020a], as well as Sims [2003]; Matějka and McKay [2015], we adopt an information theoretic form of bounded rationality and model workers and employers as facing a decision problem of choosing an action from a finite set of actions $A \in \mathcal{A}$ conditional on a payoff $u[A, \omega] : \mathcal{A} \to \mathbb{R}$ and mixed strategy $f[A|\omega] : \mathcal{A} \times \mathbb{R} \to (0, 1)$ which is a function of the wage ω. Given the payoff of each type of agent for choosing an action, there is a mixed strategy that maximizes expected payoff subject to a minimum constraint on the informational entropy of the mixed strategy, which implies rational inattention behavior [Sims, 2003]. As shown in Scharfenaker and Foley [2017] the entropy-constrained payoff-maximizing mixed strategy can also be viewed as maximizing the entropy of the mixed strategy distribution subject to a minimum constraint on expected payoff, a dual formulation that implies satisficing bounded rationality [Simon, 1956]. In both cases the frequency of actions conditional on the payoff takes the Gibbs form:

$$f[A|\omega] = \frac{e^{\frac{u[A,\omega]}{T}}}{\sum_{\mathcal{A}} e^{\frac{u[A,\omega]}{T}}} \tag{1}$$

With two actions $\mathcal{A} = \{a, \bar{a}\}$, the Gibbs distribution reduces to the logistic quantal response function expressed as a difference in payoffs:

$$f[a|x] = \frac{e^{\frac{u[a,\omega]}{T}}}{e^{\frac{u[a,\omega]}{T}} + e^{\frac{u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{u[a,\omega]-u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{2}$$

$$f[\bar{a}|\,\omega] = 1 - f[a|\omega] = \frac{1}{1 + e^{\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{3}$$

These behavioral functions are characterized by the parameter T, measured in the same units as payoff, $u[a, \omega]$, which represents the scale of "just noticeable differences" in payoffs to which individual behavior responds.[1]

An intuitive way of understanding entropy constrained behavior and Eq. 1 is in terms of the exploration-exploitation tradeoff used in reinforcement learning [Schwartenbeck et al., 2013]. When an agent faces a problem of maximizing expected utility in a complex decision-making environment pursuing actions that maximize the value of expected utility corresponds to exploitation of the environment whereas exploration corresponds to visiting or sampling alternative states. The behavioral temperature T captures the informational tradeoff associated with exploitation and exploration of rugged decision landscapes [Miller, 2016].

3 The "Shape-Up" Economy

A simplified treatment of a labor market is one in which all labor market interactions occur routinely per unit time interval. Peter Doeringer and Michael Piore Doeringer and Piore [1971] refer to this setting as a *shape-up economy*. In the shape-up economy there is a pool of unemployed workers that contract their labor for a fixed amount of time, for example one day, for a wage ω, and at the end of the day the worker returns to the original unemployed state. Employers choose to either offer employment for a given wage, or to not make an offer, in which case the work is put off for that time period. Workers can either accept the job at the offered wage, in which case they are employed for that time period and work is done, or not accept the job and remain unemployed. In each period of time the same interaction between workers and employers repeats.

3.1 Workers in the Shape-Up Economy

We can model the typical worker's action set in the shape-up economy as either *accepting* or *turning down* an offer of employment for a given wage ω: $\mathcal{A}_w = \{a_w, \bar{a}_w\} = \{\text{accept}, \text{turn down}\}$. The payoff functions for workers in the shape-up economy differ by each action. When a

[1]See Appendix A for proofs.

worker accepts a job their payoff is the money wage they receive, ω, minus the costs of working and finding a job m_w. The payoff for turning down a job is the workers fallback position z_w.

$$u_w[a_w, \omega] = \omega - m_w \tag{4}$$

$$u_w[\bar{a}_w, \omega] = z_w \tag{5}$$

$$\Delta u_w[A_w, \omega] = \omega - (m_w + z_w) = \omega - \mu_w \tag{6}$$

We refer to the total cost of not working $\mu_w = m_w + z_w$ as the “indifference wage.” With these money-equivalent payoffs workers’ quantal response distributions become:

$$f_w[a_w|\omega] = \frac{1}{1 + e^{-\frac{\Delta u_w[A_w,\omega]}{T_w}}} = \frac{1}{1 + e^{-\frac{\omega - \mu_w}{T_w}}} \tag{7}$$

$$f_w[\bar{a}_w|\omega] = 1 - f_w[a_w|\omega] = \frac{1}{1 + e^{\frac{\omega - \mu_w}{T_w}}} \tag{8}$$

The odds of a worker accepting an offer conditional on the wage are:

$$\frac{f_w[a_w|\omega]}{f_w[\bar{a}_w|\omega]} = e^{\frac{\Delta u_w[A_w,\omega]}{T_w}} = e^{\frac{\omega - \mu_w}{T_w}} \tag{9}$$

These equations tell us that the probability that a worker accepts a job offer is conditional on the difference between the offered wage ω and the indifference wage μ_w. While μ_w can be understood conventionally as a “reservation wage” in the context of this model it represents the wage at which a worker accepts a job with a probability of 50%. Only in the limit as $T \to 0$ will $f_w[a_w|\omega] = \delta[\omega - \mu_w]$, where δ is the Dirac-delta function, and μ_w correspond to the reservation wage above which workers accept employment with certainty.

3.2 Employers in the Shape-Up Economy

Employers in the shape-up economy face the quantal decision to *offer* or *not offer* employment for a given wage ω, which we can model as the action set $\mathcal{A}_c = \{a_c, \bar{a}_c\} = \{\text{offer}, \text{not offer}\}$. For employers the payoff is the difference between the marginal revenue product they receive from the worker, r_c, minus the cost of the worker, ω, and any other hiring costs, such as search costs, m_c. If an employer fails to hire their fallback position is z_c.

$$u_c\left[a_c, \omega\right] = r_c - \omega - m_c \tag{10}$$

$$u_c\left[\bar{a}_c, \omega\right] = z_c \tag{11}$$

$$\Delta u_c\left[A_c, \omega\right] = -\omega + r_c - m_c - z_c = -\omega - \mu_c \tag{12}$$

The total non-wage costs to the employer define the employer's indifference wage $\mu_c = m_c + z_c - r_c$. With these money-equivalent payoffs the conditional frequencies defining employers' actions are:

$$f_c\left[a_c \middle| \omega\right] = \frac{1}{1 + e^{-\frac{\Delta u_c[A_c, \omega]}{T_c}}} = \frac{1}{1 + e^{\frac{\omega - \mu_c}{T_c}}} \tag{13}$$

$$f_c\left[\bar{a}_c \middle| \omega\right] = \frac{1}{1 + e^{-\frac{\omega - \mu_c}{T_c}}} \tag{14}$$

The odds of an employer offering employment conditional on the wage are:

$$\frac{f_c\left[a_c \middle| \omega\right]}{f_c\left[\bar{a}_c \middle| \omega\right]} = e^{\frac{\Delta u_c[A_c, \omega]}{T_c}} = e^{-\frac{\omega - \mu_c}{T_c}} \tag{15}$$

3.3 Transaction Frequencies

An employment transaction occurs when an employer offers to hire a worker at some wage and the worker accepts the offer. Workers cannot hire themselves nor can employers produce without workers. Because each agent only controls one side of the interaction the product of the conditional action frequencies $f_c[\text{offer}|\omega]$ and $f_w[\text{accept}|\omega]$ is the probability of an employment transaction at a given wage.[2]

$$\tau[\omega] = f_c[a_c|\omega] f_w[a_w|\omega] = \frac{1}{\left(1 + e^{-\frac{\omega - \mu_c}{T_c}}\right)\left(1 + e^{-\frac{\omega - \mu_w}{T_w}}\right)} \tag{16}$$

Figure 1 shows the logit quantal response curves for a worker and employer and the transaction probability as a function of the wage ω.

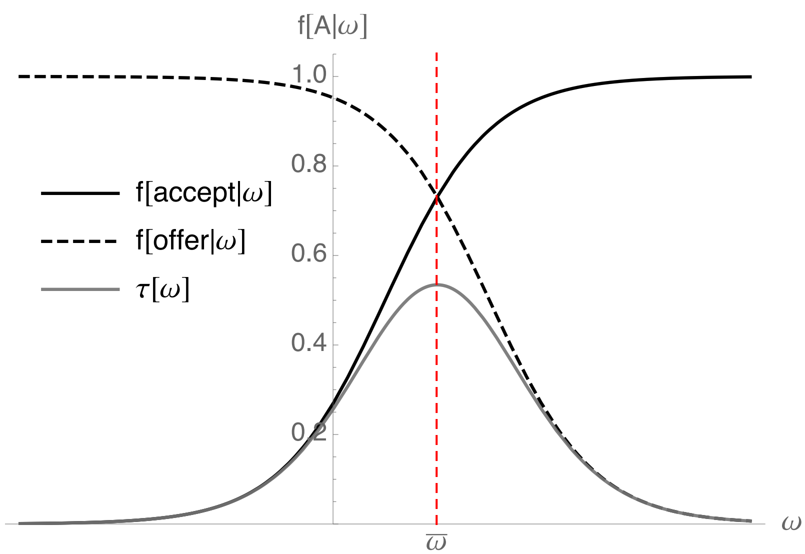

Figure 1: Logit quantal response curves for workers and employers and transaction frequencies for $\mu_w = 0.5, \mu_c = 1.5$ and $T_w = T_c = 0.5$.

The parameters μ_w and μ_c are the indifference points at which a typical worker would accept a job with a 50% probability and a typical employer will make an offer with a 50%

[2]Employers can offer employment that a worker turns down, but this probability is not the probability of becoming unemployed. In keeping with conventional measures of unemployment we do not consider workers who do not accept a job that is offered as unemployed.

probability. The wage at which the quantal response frequencies of workers and employers are equal is the sum of these "indifference prices" weighted by the relative behavioral temperatures:

$$\omega^* = \mu_c \frac{T_w}{T_c + T_w} + \mu_w \frac{T_c}{T_c + T_w} \tag{17}$$

At the intersection of the labor offer and employment offer curves the transaction frequency is:

$$\tau\left[\omega^*\right] = \frac{1}{\left(1 + e^{\frac{\mu_w - \mu_c}{T_c + T_w}}\right)^2} \tag{18}$$

In the special case when both agents have identical behavioral parameters $T_w = T_c$, the transaction frequency distribution is symmetric and the wage at the average transaction frequency is $\omega^* = \frac{\mu_c + \mu_w}{2}$. These conditions approximate a supply-demand equilibrium as the mean wage is the average of the two agents' indifference wages.

4 Unemployment and Job Vacancies

Unemployment in the shape-up economy consists of all workers who are not hired in a period. While the shape-up economy is a good description of the type of informal markets that exist in the parking lots of Home Depot, the broader *job economy* is comprised of workers with both definite and indefinite job tenure. A job economy, however, can be described by the model of a shape-up economy if we assume job tenure is an exogenous variable that is independent of the wage.

The frequency with which a worker will see an offer of employment at any wage will depend on the number of employers per worker, and the average number of vacancies each employer attempts to fill by making job offers each period. Assuming the probability of job separation in any period, p, is exogenous, then $t = 1/p$ is the average job tenure.

The total number of jobs is $K = V + F$ where V is the number of vacant job openings and F is the number of filled jobs. There are M firms and a labor force $L = N + U$ comprised of N employed workers and U workers available to work. The number of openings relative to the total number of jobs is the job vacancy rate $v = V/K$. The ratio of unemployed workers to the labor force is the unemployment rate, $u = U/L$, while $n = N/L$ is the employment rate, and $k = K/L$ defines the job/worker ratio. The number of filled jobs is equal to the number of employed workers which implies the standard identities:

$$K - V = F = N = L - U \tag{19}$$

$$(k - v) = n = (1 - u) \tag{20}$$

$$(1 - v)k = (1 - u) \tag{21}$$

Vacancies and unemployment increase as employed workers are separated from their job, which happens at a constant rate of p, so that the number of workers becoming unemployed is pN. The change in unemployment decreases when unemployed workers find employment, which is the probability of a worker accepting employment conditional on a wage times the probability of an employer offering employment conditional on a wage times the frequency of workers for a given wage $\int \tau[\omega] f[\omega] d\omega$.

$$\Delta U = pN - U \int \tau[\omega] f[\omega] d\omega \tag{22}$$

$$\Delta U = p(1 - u)L - uL \int \tau[\omega] f[\omega] d\omega \tag{23}$$

In equilibrium $\Delta U = 0$ and the unemployment rate is

$$u = \frac{p}{p + \int \tau[\omega] f[\omega] d\,\omega} \tag{24}$$

Similarly, the change in job vacancies increases with job separations, pN and decreases by the number of workers who find employment with probability $\tau[\omega]$ at wage at wage $f[\omega]$.

$$\Delta V = pN - V \int \tau[\omega] f[\omega] d\,\omega \tag{25}$$

$$\Delta V = p(1 - v)kL - vL \int \tau[\omega] f[\omega] d\,\omega \tag{26}$$

In equilibrium $\Delta V = 0$ and the job vacancy rate is

$$v = \frac{kp}{kp + \int \tau[\omega] f[\omega] d\,\omega} \tag{27}$$

In the United States, the job to worker ratio k tends to be close to but below unity and the average job tenure is between close to four years, or approximately 48 months, making $p = 1/48$.

5 The Wage Distribution

The assumption of quantal responses in actions tends to induce a strong correlations between the outcome and actions. For example, because workers are more likely to accept a job offer at a high wage, the quantal response effect will tend to produce a higher worker expected wage conditional on accepting than the worker expected wage conditional on rejecting an offer. Thus, workers' actions and wages tend to be positively correlated. Similarly, because employers are more likely to offer jobs at lower wages, the quantal response effect will tend to produce a lower employer expected wage conditional on offering than the employer expected wage conditional on not making an offer. Thus, employers' actions and wages tend to be negatively correlated.

In the absence of further constraints maximizing entropy of the joint distribution $f[\omega, A_w, A_c]$ will tend to maximize the differences of expected wages conditional on actions for each agent. In market interactions, however, these correlations are offset by the impact of the action on the outcome. For example, when a worker accepts a job offer that tends to lower the wage for that job, and when a worker rejects an offer it tends to raise the wage for that job. Similarly, when an employer offers a job, that tends to raise the wage for the job and when an employer refrains from making an offer, that tends to lower the wage for that job. To reflect this feedback or impact effect in the constrained maximum entropy framework [Jaynes, 1983], we limit the differences in worker expected wages conditional on accepting and rejecting offers, and the parallel differences in employer expected wages conditional on making and not making an offer, in both cases tending to move the wage relative to an exogenous market-determined level, α, which is common to both agents since they are interacting in the same market.

$$f_w\left[a_w\right] E\left[\omega-\alpha \left| a_w\right.\right]-f_w\left[\bar{a}_w\right] E\left[\omega-\alpha \left|\bar{a}_w\right.\right] \leq \delta_w \tag{28}$$

$$f_c\left[\bar{a}_c\right] E\left[\omega-\alpha \left|\bar{a}_c\right.\right]-f_c\left[a_c\right] E\left[\omega-\alpha \left| a_c\right.\right] \leq \delta_c \tag{29}$$

Even though worker decisions to reject offers and employer decisions to refrain from making offers are not directly observable in data on wages these constraints are theoretically meaningful and in principle reflect real impacts of decisions on wage levels. Plugging in the quantal response functions we find:

$$\int \tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] f[\omega](\omega-\alpha)d\,\omega \leq \delta_w \tag{30}$$

$$\int \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right] f[\omega](\alpha-\omega)d\,\omega \leq \delta_c \tag{31}$$

Because workers and employers interact in the same market we can simplify the model by writing this constraint as a single equation:

$$\int \left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\right) f[\omega](\omega-\alpha)d\,\omega \leq \delta \tag{32}$$

This constraint can also be expressed as the difference of the odds of workers accepting and employers offering weighted by the transaction frequencies because

$$2\left(e^{-\frac{\Delta u_c[A_c,\omega]}{T_c}} - e^{\frac{\Delta u_w[A_w,\omega]}{T_w}}\right)\tau[\omega] = \tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right] \tag{33}$$

The statistical equilibrium distribution maximizes the entropy of the joint distribution $f\left[\omega, A_w, A_c\right]$ subject to the normalization of probabilities and the feedback constraint:

$$\underset{f[\omega]\geq 0}{\text{Max}} - \int \sum_{\mathcal{A}_w}\sum_{\mathcal{A}_c} f\left[\omega, A_w, A_c\right] \text{Log}\left[f\left[\omega, A_w, A_c\right]\right] d\omega \tag{34}$$

$$\text{subject to} \int \left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\right) f[\omega](\omega-\alpha)d\,\omega \leq \delta \tag{35}$$

$$\text{and} \int \sum_{\mathcal{A}_w}\sum_{\mathcal{A}_c} f\left[\omega, A_w, A_c\right] d\omega = 1 \tag{36}$$

We can write the maximum entropy problem in terms of the marginal and conditional distributions and solve for the marginal frequencies of the wage.[3]

$$\underset{f[\omega]\geq 0}{\text{Max}}\, H = -\int f[\omega]\text{Log}[f[\omega]]d\omega + \int f[\omega] H\left[f\left[A_w|\,\omega\right]\right] d\omega + \int f[\omega] H\left[f\left[A_c|\,\omega\right]\right] d\omega \tag{37}$$

$$\text{subject to} \int \left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\right) f[\omega](\omega-\alpha)d\,\omega \leq \delta \tag{38}$$

$$\text{and} \int f[\omega]\, d\omega = 1 \tag{39}$$

The first-order conditions are sufficient to characterize a unique solution:

$$f[\omega] = \frac{e^{H[f_w[A_w|\omega]]} e^{-\left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right]\frac{(\omega-\alpha)}{S}\right)} e^{H[f_c[A_c|\omega]]} e^{-\left(\tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\frac{(\omega-\alpha)}{S}\right)}}{\mathbb{Z}[\omega; u_w, u_c, T_w, T_c, S, \alpha]} \tag{40}$$

where $\mathbb{Z}$ is the normalizing constant. Plugging in the payoff functions 6 and 12 the statistical equilibrium wage distribution is proportional to the product of both agent's QRSE distributions:

$$f[\omega] \propto e^{H[f_w[A_w|\omega]]} e^{-\tanh\left[\frac{\omega-\mu_w}{2T_w}\right]\frac{(\omega-\alpha)}{S}} e^{H[f_c[A_c|\omega]]} e^{-\tanh\left[\frac{\omega-\mu_c}{2T_c}\right]\frac{(\omega-\alpha)}{S}} \tag{41}$$

where $\alpha, \mu_c, \mu_w \in \mathbb{R}$ and $T_c, T_w, S \in (0, \infty)$. In this model α is the market location or inflection point of the feedback of actions on the outcome, μ_c, μ_w are behavioral locations which are the inflection points of indifference between taking one action or another, S represents the market temperature or scale of the feedback, and T_c, T_w represent the behavioral temperature or scale of the behavioral responses.

[3] See Appendix A for proofs.

6 Unfulfilled Expectations

Equilibrium in expectations is defined by Phelps [1994] as the state in which expectations of market partipants are fulfilled. When expectations of participants are not fulfilled, they will in general face market-based penalties to their actions which will incentivize agents to change their behavior. An important consequence of such behavioral changes in a system comprised of interacting entropy-constrained participants is that the state of the system will also change in response to changes in individual behavior Foley [2020b]. Figure 2 represents labor market equilibrium with fulfilled expectations. In this situation employers and workers will not revise their expectations absent any unanticipated shocks to the system. Equilibrium unemployment is purely frictional and arises because of the decentralized nature of market interactions common in the search theoretic literature Diamond [1982].

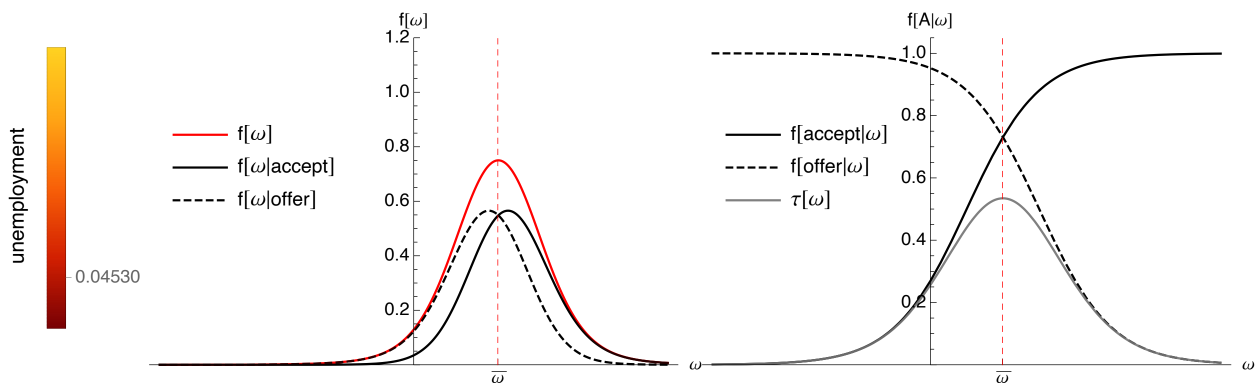

Figure 2: Labor market equilibrium wage distribution and conditional action frequencies for $\mu_w = 0.5, \mu_c = 1.5$, $T_w = 0.5, T_c = 0.5$, $\alpha = 1$, and $S = 1$. In this situation expectations are fulfilled and the average market wage $\bar{\omega} = \frac{\mu_c + \mu_w}{2}$.

Figure 2 approximates supply-demand equilibrium in labor market transactions as the modal and mean wage is the average of the two agents' valuations of work.

Now consider an exogenous negative shock to the economy, which may be a change in the money supply, or decline in aggregate demand. Such a shock initially only changes employers' willingness to hire as represented in Figure 3 as a 30% decline in μ_c. In this situation unemployment increases by 1.6% to approximately 6.2% and is now due to a combination of frictional unemployment, which always exists in decentralized interactions, and involuntary unemployment, which arises due to the unfulfilled expectations of workers.

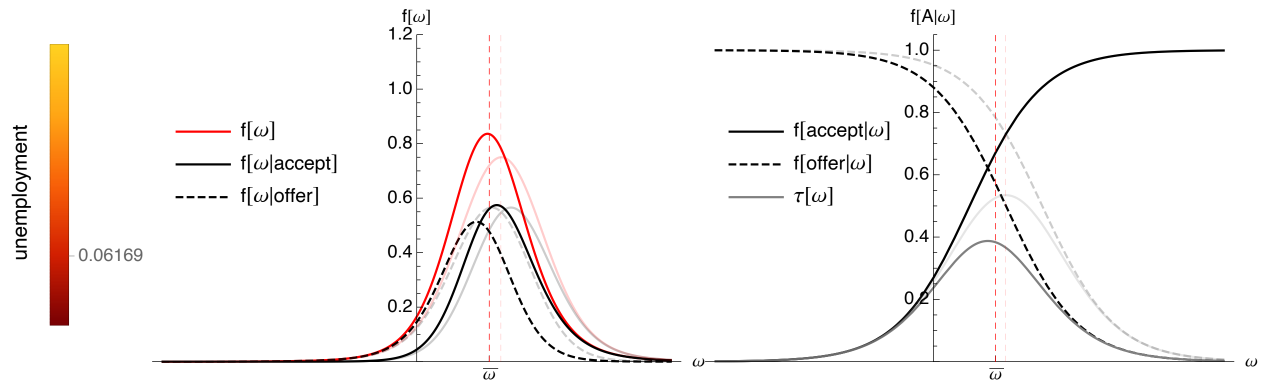

Figure 3: Labor market equilibrium wage distribution and conditional action frequencies after a 30% shock to employers' indifference wage: $\mu_w = 0.5, \mu_c = 1$, $T_w = 0.5, T_c = 0.5$, $\alpha = 1$, and $S = 1$. In this situation expectations are unfulfilled and the average market wage $\bar{\omega}$ is below the intersection of action frequencies.

If the shock is permanent there will be a proportional adjustment of workers' expectations and a relocation of the market, as captured by a decline in the parameters μ_w and α. Figure 4 demonstrates that at the new equilibrium expectations are once again fulfilled, albeit at a new lower equilibrium average market wage. Equilibrium unemployment, however, does not reassert itself at the pre-shock equilibrium rate of 4.5% as one might expect if money was neutral in the long run. Instead, the new equilibrium is defined by both a lower average wage and a higher rate of unemployment.

Unless there is a proportional decline in the behavioral and market scale parameters T_w, T_c and S the new equilibrium unemployment will be higher after agents realign their expectations with the market. Figure 5 shows that only when agents' behavioral responses and the market feedback response decline in proportion to the shock will unemployment be neutral to the shock.

An important implication of the statistical equilibrium perspective is that the corrections of expectations by market participants leads to inertia in the adjustment of the system. In contrast the rational expectations, which assumes that such adjustments are instantaneous

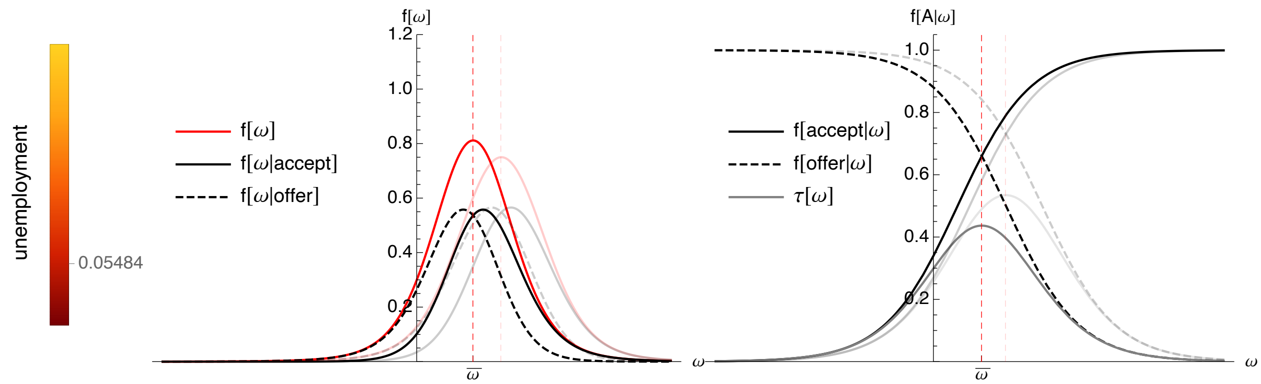

Figure 4: Labor market equilibrium wage distribution and conditional action frequencies after a proportional decline in workers' expectations and market location: $\mu_w = 0.33, \mu_c = 1$, $T_w = 0.5, T_c = 0.5$, $\alpha = 0.66$, and $S = 1$. In this situation expectations are again fulfilled, but at an average market wage lower than before the shock and at a higher equilibrium rate of unemployment.

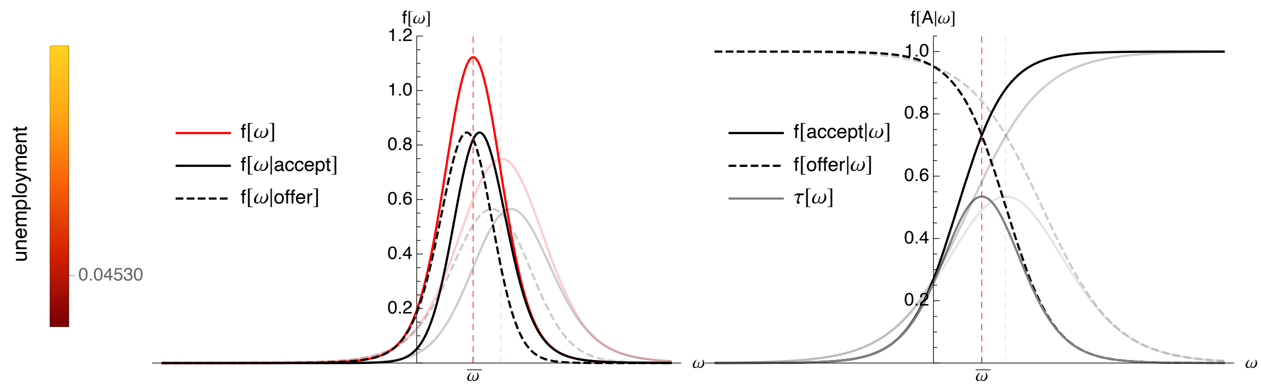

Figure 5: Labor market equilibrium wage distribution and conditional action frequencies after a proportional decline in workers' expectations, market location, and behavioral and market temperatures: $\mu_w = 0.33, \mu_c = 1$, $T_w = 0.33, T_c = 0.33$, $\alpha = 0.66$, and $S = 0.66$. In this situation expectations are fulfilled and the rate of unemployment has adjusted to the pre-shock equilibrium rate due to the proportional decline in scale factors T_w, T_c, S.

and costless, expectations with informational entropy constrained participants can not be identified with actual market outcomes. Because the hypothesis of rational expectations [Friedman, 1968; Muth, 1961] has no analog to the behavioral and market temperatures (which are always implicitly taken to be zero) the equilibrium rate of unemployment is independent of instantaneous adjustments of the system to new states.

7 Discussion

The discussion of the neutrality of money as far as we know has up to this point rested on assumptions of the homogeneity of demand and supply curves with respect to the level of prices and wages. This assumption has led economists to the view that a change in a nominal exogenous variable, such as the money supply or aggregate demand, is analytically equivalent to a change in the denomination of the currency. Under this assumption the neutrality of money in the sense of invariance of real outcomes to changes in the money supply or aggregate demand is guaranteed. Classic analyses of the neutrality of money such as Friedman [1968] invariably invoke this principle.

The constrained-entropy form of bounded rationality we assume in the current stylized model of the labor market, however, underlines the importance of another dimension to this question. When currencies are re-denominated, for example, the euro replacing the franc, it is not implausible to suppose that employers and workers adjust both the level of their expected offers and their expectations of the scale of fluctuations in offers proportionately, which, as we have seen in our model, would lead to the same "real" outcome in terms of wage levels and unemployment. But when aggregate demand changes due to a shock in monetary policy or the broader economic environment, the scale on which employers and workers judge differences in wage offers may not adjust at the same rate as their expectations of the level of wage offers. As we see in the current model, this type of uneven adjustment can lead to real changes in the wage and level of unemployment. Milton Friedman often alluded to "the level of unemployment that would be ground out by the Walrasian system," but neither the Walrasian system nor Friedman's own models of labor market equilibrium addressed the dimension of reactions to fluctuations in wage offers. But we can see from the current model that it is precisely this dimension, represented by the "temperature-like" scale factors T_w, T_c, S that "grinds out" equilibrium levels of frictional unemployment.

7.1 Efficiency Wages

The statistical equilibrium model of labor market interactions is also consistent with the existence of efficiency wages. If there is a positive bid-ask spread in the wage between employers and workers then the average wage will be above workers' indifference wage. This gap can motivate worker effort through a threat of dismissal. There is an additional perceived cost to job loss that disciplines workers because once unemployed there is a non-zero probability of remaining unemployed. The important difference between the conventional theory of efficiency wages and the statistical equilibrium wage distribution is that in statistical equilibrium the gap between the average wage and indifference wage is not an explicit policy of employers. Thus, while the statistical equilibrium model produces a positive gap between workers' indifference wage and the average wage, it does so independently of employers' intentions. In this sense, the efficiency wage arises in statistical sense as an unintended consequence of labor market interactions.

8 Conclusions

Social outcomes can arise from complex market interactions with non-negligible feedbacks that stabilize the state variables into equilibrium frequency distributions. Entropy constrained behavior representing the endogenous uncertainty of individual agents interacting through social and economic institutions cast new light on core problems of macroeconomics including wage fluctuations and involuntary unemployment. The quantal response statistical equilibrium distribution is a parsimonious description of the endogenous fluctuations and higher moments of the macroeconomic state variable. Shocks to the system change the parameters of the QRSE distribution that can be easily understood in terms of individual- and system-level behavior. The incorporation of behavioral and market scale parameters help to explain endogenous fluctuations in observable macroeconomic phenomena and the role of unfulfilled expectations in determining changes in the state of system. In a labor-market setting the QRSE model clarifies the causes and differences in frictional and involuntary unemployment.

Appendix A: Proofs

Entropy-Constrained Behavior

We assume that workers and employers choose an action from a finite set of actions $A \in \mathcal{A}$ with an associated payoff $u[A, \omega] : \mathcal{A} \to \mathbb{R}$ and maximize their expected payoff subject to a minimum constraint on the entropy of the mixed strategy:

$$\underset{f[A|\omega]\geq 0}{\text{Max}} \sum_{\mathcal{A}} f[A|\omega]u[A, \omega] \tag{A.1}$$

$$\text{subject to} \sum_{\mathcal{A}} f[A|\omega] = 1 \tag{A.2}$$

$$\sum_{\mathcal{A}} -f[A|\omega] \log f[A|\omega] \geq H_{\min}[\omega] \tag{A.3}$$

The Lagrangian associated with this programming problem is:

$$\begin{aligned} \mathcal{L}[f; \lambda, T] = & -\sum_{\mathcal{A}} f[A|\omega]u[A, \omega] - \lambda \left(\sum_{\mathcal{A}} f[A|\omega] - 1 \right) \\ & + T \left(\sum_{\mathcal{A}} f[A|\omega] \log[f[A|\omega]] - H_{min} \right) \end{aligned} \tag{A.4}$$

The first-order conditions require the conditional action frequencies to be distributed according to the Gibbs distribution:

$$f[A|\omega] = \frac{e^{\frac{u[A,\omega]}{T}}}{\sum_{\mathcal{A}} e^{\frac{u[A,\omega]}{T}}} \tag{A.5}$$

This problem has the dual form of maximizing the entropy of the mixed strategy subject to normalization of probabilities and a minimum expected payoff representing "satisficing" bounded rationality:

$$\underset{f[A|\omega]\geq 0}{\text{Max}} \sum_{\mathcal{A}} -f[A|\omega] \log f[A|\omega] \tag{A.6}$$

$$\text{subject to} \sum_{\mathcal{A}} f[A|\omega] = 1 \tag{A.7}$$

$$\sum_{\mathcal{A}} f[A|\omega] u[A,\omega] \geq U_{min}[\omega] \tag{A.8}$$

In this case the first-order conditions require

$$f[A|\omega] = \frac{e^{\beta u[A,\omega]}}{\sum_{\mathcal{A}} e^{\beta u[A,\omega]}} \tag{A.9}$$

The Lagrange multiplier T is the entropy cost of increasing expected payoff, or the terms on which the agent trades off information and expected payoff. The Lagrange multiplier $\beta = 1/T$ is the inverse of the behavior temperature T and has the behavioral interpretation of the expected payoff cost of increasing entropy, or the terms on which the agent trades off expected payoff and information. With two actions $\mathcal{A} = \{a, \bar{a}\}$, the Gibbs distribution reduces to the logistic quantal response function:

$$f[a|\omega] = \frac{e^{\frac{u[a,\omega]}{T}}}{e^{\frac{u[a,\omega]}{T}} + e^{\frac{u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{u[a,\omega]-u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{A.10}$$

$$f\left[\bar{a}|\,\omega\right] = 1 - f[a|\omega] = \frac{1}{1 + e^{\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{A.11}$$

Joint, Marginal, and Conditional Entropy

The joint entropy can be written in terms of the marginal and conditional entropies:

$$H\left[\omega, A_w, A_c\right] = H[\omega] + H[A_w|\omega] + H[A_c|\omega] \tag{A.12}$$

where we assume that $H\left[A_w|A_c\right] = H\left[A_w\right]$ so that workers' and employers' decisions are conditionally independent of one another. This assumption only implies that workers and capitalists only interact through the wage ω. The entropy of the actions of workers and employers conditional on the wage is

$$H\left[A_w|\,\omega\right] = \int f[\omega] H\left[f\left[A_w|\,\omega\right]\right] d\omega \tag{A.13}$$

$$H\left[A_c|\,\omega\right] = \int f[\omega] H\left[f\left[A_c|\,\omega\right]\right] d\omega \tag{A.14}$$

The joint entropy can be written as:

$$H\left[\omega, A_w, A_c\right] = -\int f[\omega]\mathrm{Log}[f[\omega]]d\omega + \int f[\omega] H\left[f\left[A_w|\,\omega\right]\right] d\omega + \int f[\omega] H\left[f\left[A_c|\,\omega\right]\right] d\omega \tag{A.15}$$

where $H\left[f_i\left[A_i|\,\omega\right]\right] = -\left(\frac{1}{1+e^{\frac{\Delta u_i[A_i,\omega]}{T_i}}}\mathrm{Log}\left[\frac{1}{1+e^{\frac{\Delta u_i[A_i,\omega]}{T_i}}}\right] + \frac{1}{1+e^{-\frac{\Delta u_i[A_i,\omega]}{T_i}}}\mathrm{Log}\left[\frac{1}{1+e^{-\frac{\Delta u_i[A_i,\omega]}{T_w}}}\right]\right)$ for $i = \{w, c\}$ is the entropy of the conditional action function for workers and employers.

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