The Economics of Pensions
Remarks on Growth, Policy and Class Conflict

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Abstract
This paper compares fully-funded (FF) and pay-as-you-go (paygo) pension plans in a Keynesian framework for an economy with overlapping generations and excess capacity. The model addresses both short/medium-run equilibria and steady-states. Income distribution and class conflict, two crucial aspects of the political economy of pensions, become multi-dimensional. In a fully-funded economy class conflict between capitalists and labor gets diffused in the short-run by retirees’ own interest to maintain a high profit share. In the long-run capitalists recognize that they can control their net share of profits by controlling employment and therefore the number of future retirees through capital accumulation. An extension of the model can show that fiscal policy is not always helpful in a fully-funded economy. A pay-as-you-go economy maintains a closer resemblance to the classical story of class conflict over income distribution. This is because workers and retirees have their interests aligned with the wage share. In this case fiscal policy through spending can be effective without creating a debt problem.

Keywords: private accounts; social security; Keynesian economics; distributive conflict;
JEL Classification: E24; E12; G23; H55;

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1 Introduction

Population aging in the industrialized world is expected to accelerate over the next several decades complicating the global development process. A prevalent view

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among politicians, policy makers and academics alike, especially in the US, is that a change in the status quo of intergenerational transfers is inevitable. An attack is being waged against public goods and services provision to elderly such as the Medicare program in the US, and against defined-benefit pension schemes, of which the public-run social security system is the most prominent. I claim that arguments in favor of privatizing public pensions and/or moving towards a defined-contribution system and shifting provision of some goods and services to the private sector are often wrong because they are based on misleading theoretical models that ignore issues of distribution and class conflict. In addition, many of the assumptions of the typical neoclassical overlapping generation model (hereafter OLG) do not reflect the realities of most economies nor how individuals think and act when it comes to planning for retirement. Saving-driven investment or labor supply constraints are examples of these assumptions at the macroeconomic level. It remains true however that the neoclassical OLG model continues to inform the public and policy on how to address challenges posed by an increase in the share of elderly in the economy.\(^1\) The implications can be costly not only for the economic development process but also for the social and human welfare.

This paper contributes to the non-mainstream literature by providing an alternative to the neoclassical OLG model.\(^2\) Specifically, and first, I give the distributive conflict a prime place in the analysis of dynamics of economies and their pension plans. I investigate two dimensions of income distribution: in the primary stage income is distributed between active factors of production who receive profits and wages; during the second stage, transfers are made to retired households and hence we get a change in net shares of income. Secondly, aggregate demand plays a role in the determination of economic activity. Thirdly, investment and saving decisions are disconnected and as a result firms decision to accumulate capital is based on other factors than the available saving. The device at work in this paper is an accounting consistent macro model for an economy with excess labor supply and capacity utilization, and an independent investment demand. The approach and modeling choices follow the typical Keynesian-Kaleckian framework that appears in Taylor and Lysy (1979), Taylor (1985), Taylor (1991), and Taylor (2004). Next, I use the model to examine and compare defined-benefit or pay-as-you-go and defined-contribution or fully-funded pensions plans. Various comparative dynamics are conducted and the basic model is extended to take account of policy intervention.

## 2 Some insights

The economics of pensions and population gets complicated by the need to record changes in the variables we work with in historical time. I make several simplifying

\(^1\)As Michl and Foley (2004) point out projections on the fiscal solvency of the US Social Security System are based on the framework provided by the Solow-Swan growth model.

assumptions here. The first one is that a person works in the first period and retires in the second period. Employment in a given period $t$ is given by:

$$L_t = bX_t$$  \hspace{1cm} (1)

where $b = L/X$ is the labor-output ratio, which, for now, we hold constant, and $X_t$ is output. Last period’s employment can be written as $L_{t-1} = bX_{t-1}$. Since last period’s workers become the current retirees, we can write the old-age dependency rate, $d = R/L$ as:

$$\frac{R_t}{L_t} = \frac{X_{t-1}}{X_t} = \frac{1}{g_t + 1}$$  \hspace{1cm} (2)

The insight from (2) is that the dependency rate depends on the rate of expansion of the economy, $g_t$. But it is also true that the number of tomorrow’s retirees ($n_{r,t+1}$), who are today’s workers, is itself determined by today’s growth. Replacing $L_t$ by $R_{t+1}$ in (2) we get:

$$n_{r,t+1} = g_t$$  \hspace{1cm} (3)

In a system where current workers support yesterday’s workers it is clear based on (3) that an economy with a better past performance could face difficulties in meeting its obligations towards its current retirees. In a sense, a crisis of a social security system can be induced not necessarily by a demographic transition such as the famed baby-boomer story in the US, but simply by a prolonged recession.

A private pension system based on a defined-contribution scheme is thought to avoid this problem since current workers have no ”obligations” towards current retirees. This is not equivalent with retirees being insulated from what the current economy is, or is not achieving. A poor performance implies lower profit rates and therefore lower income for retirees. For its supporters, the private pension plan is equivalent to an investment decision taken by a rational economic agent for the goal of future returns. In reality however a pension is closer to an insurance that the worker acquires against the risk of living long enough to become old and unable to work. A failure of the economy to deliver once this individual risk materializes is therefore a failure of the economy to meet its obligations. Legally, of course, that does not pose an issue since defined-contribution pension accounts do not carry an obligation by the managing entity of the pension schemes to provide a certain level of annual income or pension annuity. And this, indeed, is the issue at hand. The risk is being privatized for some of the most vulnerable groups in the society with dire consequences as many have discovered in the aftermath of the 2008 financial collapse.

Two scenarios are possible when we consider population growth dynamics. If labor force grows faster than the economy $n > g = \hat{L}$ we get an ever increasing unemployment and no effects on the pension system.\footnote{This is the case we may add if wages are independent of the employment rate.} If labor force expands at a lower rate than output, the economy eventually exhaust all its labor reserves and
becomes constrained by the availability of labor or \( g_t = n_t \). Expression (2) is then:

\[
R_t = \frac{L_t}{n_t + 1}
\]  

(4)

Based on (4) a decline in the growth rate of working-age population, \( n_t \), can indeed raise dependency rates and cause problems for a social security system. This conclusion holds only if we assume that output growth is determined by the available supply of labor and that productivity remains constant, a situation that I do not consider in this paper. At the steady-state it remains the case that the economy, its population, employment and retired population all grow at the same rate \( n \).

3 National accounting and model setup

Generally, two main pension systems provide the bulk of old-age income in the developed economies. The paygo collects taxes from current workers and disburses them to current retirees. A fully-funded system receives contributions from current workers, invests them on their behalf, and returns the principal and the interest income earned as annuities once the worker has retired. We start with a description of the national accounting and equilibrium relation for an economy with paygo and FF pension systems. Next, I derive the accounting relations for primary and secondary distribution of income. The section concludes with few exercises in comparative statics.

3.1 Accounting in a paygo economy

To keep things simple let’s assume that in the PAYGO system the taxes collected from workers are transferred entirely to retirees. In reality – the US is an example – the surplus of social security contributions over current disbursements to retirees are held in the Social Security Trust Fund which invests its available funds in US Treasury securities only. Let the tax rate be given by:

\[
\rho_t = \frac{pR_t}{wL_t}
\]  

(5)

where \( p, R, w \) and \( L \) are the level of pension in real terms, number of retirees, real wage and employed workers. Depending of the timeframe we are working with I assume two different scenarios for \( \rho \). The two scenarios, as will become evident in a moment, have very different implications for class conflict. In the first scenario, which describes short to medium-run dynamics, \( \rho \) is set by policy at a level \( \bar{\rho} \). The retiree’s individual income is endogenous and follows from:

\[
p = \frac{\bar{\rho}wL}{R}
\]  

(6)

If the retired population is politically weak and therefore unable to lobby an increase in \( \bar{\rho} \), the individual pension level declines as the number of retirees, \( R \), increases. In
this situation class conflict is suppressed beyond the primary distribution of income. On the other hand, if policy responds to retirees' demands for a rise in the social security tax, the after-tax wage share or workers’ disposable income declines. It can be expected that workers would bargain for higher wages. If they are successful, the primary distribution of income changes and as a result production relations change. In the second scenario and in the long-run $\rho$ follows endogenously from the accounting equation:

$$\rho = \frac{pR}{wL} \quad (7)$$

As the number of retirees increases the social security tax has to rise unless the wage level or employment goes up. In essence, robust output growth is necessary as can be observed if we rewrite (5) as:

$$\rho_t = \frac{p}{w(g_t + 1)} \quad (8)$$

Either demographic changes or, as mentioned before, a prolonged decline in the economic performance make it conceivable that policy could change in the future. If, for example, all things equal the demographic dependency rate $d = R/L$ increases the change would have to target either the benefits or income transferred to retirees $p$, the tax rate $\rho$, or income distribution $\psi = wL/X$, or a combination of all these. We work with both specifications for $\rho$ discussed here. In the static short-run version of the model $\rho$ is kept fixed. Moving on to the dynamic model and stories of the long-run $\rho$ becomes an endogenous variable.

### 3.2 Accounting in a fully-funded economy

Employees contribute $q_t$ amount to their “pension accounts” in the FF system calculated as a percentage of their current wage income $q = \tau w$. In effect this contribution buys the retiree a share of capital. Once the worker retires she receives $q(1 + r_t)$ where $r_t$ is the rate of profit. Total current contributions by the working labor are $\tau wL_t = \tau \psi X_t$, while current retirees’ benefits amount to $\tau \psi_{t-1}X_{t-1}(1 + r)$. In an economy with a FF system the share of wealth or capital held by retirees is variable of interest and, in a way, it is the counterpart of $\rho$ in a paygo scheme. Let this share be defined as $\eta_t = \tau wR_t/K_t$. Since the number of retirees is a function of last period’s income and $X_{t-1} = X_t/(g_t + 1)$ we can rewrite $\eta_t$ as:

$$\eta_t = \frac{\tau w_{t-1}bX_{t-1}}{K_t} = \frac{\tau \psi_{t-1}u_t}{g_t + 1} \quad (9)$$

where $u_t$ is capacity utilization. As before, the presence of retirees in the economy, now quantified in terms of their share of capital, is controlled by the rate of growth of the economy. In addition, a higher capacity utilization benefits $\eta$. The numerator, which is a function of retirees’ income, grows with better economic activity.
3.3 National accounting, transfers and distribution

National accounting for an economy with transfers to retirees can then be described by the following system of equations for consumption $C$ by the three economic classes: working household, retired households and capitalist households respectively.

$$C_{w,t} = (1 - \tau)(\psi X_t - \rho \psi X_t) \quad (10)$$
$$C_{r,t} = \rho \psi X_t + q R_t (1 + r) \quad (11)$$
$$C_{\pi,t} = (1 - s) (\pi X_t - q R_t r) \quad (12)$$

where $s$ and $\tau$ are marginal propensities to save of capitalists and wage-earners respectively, and $\pi = r K / X$ is the profit share. Notice that workers are assumed to save only for life-cycle and not for bequest reasons. With the entire social contributions dispensed to retirees $(\rho \psi X_t = p R_t)$, the macro equilibrium condition becomes:

$$s \pi X_t + \tau \psi X_t - q R_t s \pi u_t - \rho \tau \psi X_t = I_t + q R_t \quad (13)$$

In a Keynesian world with excess capacity a macro imbalance is corrected through a change in the economic activity. Formally, output or capacity utilization $u$ become the adjusting variables. Dividing (13) by $K$ and solving for $u$ we get:

$$u = \frac{g + \eta}{s \pi (1 - \eta) + s \psi (1 - \rho)} \quad (14)$$

where $s \pi = s \pi$ and $s \psi = \tau \psi$ are saving rates of capitalists and workers respectively. Investment demand $g$ is exogenous in the short-run, and independent of available saving in the economy. We return to (14) in a moment with few comparative statics exercises.

Meanwhile, we direct our attention to the national accounting for the distribution of income. In the first stage, national income goes to active factors of production, capital $K$ and labor $L$, according to $X = \Pi + W$. Written in primary income shares this relation becomes $1 = \psi + \pi$. Dubbed the two souls problem this simple accounting relation offers a way to introduce class conflict in the analysis of economic dynamics.

The conflict is rather obvious: an increase in the wage share can happen only at the expense of the profit share in national income.

In this paper I extend the two souls problem to include the retired population as an important player in the determination of income distribution. To do this we need to look at the secondary distribution of income which involves transfers to main economic classes. Pensions can be funded either from wages in the paygo system or from profits in the case of private pension accounts. We can re-write the national

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4 We use here the definition of the profit rate that $r = \pi u$.

5 Flaschel (2009) borrows the analogy of the two souls from Goethe’s Faust. Taylor (2011) discusses at great length the implications of an independent theory of income distribution for the analysis of macroeconomic dynamics.
income relation as:

\[ X = (1 - \rho)\psi X + pR + (1 - \eta)\pi X \]  \hspace{1cm} (15)

where \( p \) is the level of individual income of retirees funded by both pension schemes. Dividing (15) by \( X \) we get:

\[ 1 = (1 - \rho)\psi + (1 - \eta)\pi + \theta \]  \hspace{1cm} (16)

where \( \theta \) is the share of overall income of retired households in the national income, and \( \psi^N = (1 - \rho)\psi \) and \( \pi^N = (1 - \eta)\pi \) are net income shares of workers and capitalists respectively. The social conflict surrounding the distribution of income has now become multi-dimensional. Both \( \rho \) and \( \eta \) are in effect a tax on workers’ and capitalists’ income respectively. An increase in \( \theta \) is possible only through a decline in the net share of wages and/or profits. Understanding how workers and capital owners are likely to respond to a decline in their disposable income is crucial for the success of any pension reform.

3.4 Comparative statics

Few comparative statics exercises are in order now. The first thing to notice (see (14)) is that a larger presence of retirees in the economy as measured by \( \eta \) and \( \rho \) always stimulates the economy. The explanation is simple. Retirees consume everything they earn in terms of annuities or social security disbursements. Hence, strong past growth produces a larger share of retirees in the present. In a demand-constrained economy this outcome affects positively current economic performance.

A change in income distribution between profits and wages has a direct impact on aggregate demand and therefore on the equilibrium level of capacity utilization according to:

\[ \frac{du}{d\pi} = -[s(1 - \eta)] - \tau(1 - \rho)]\Theta \]  \hspace{1cm} (17)

where \( \Theta = (g + \eta)/(s\pi(1 - \eta) + s\psi(1 - \rho)) > 0 \). The partial derivative (17) is positive if \( \rho \) is small and \( \tau \) and \( \eta \) are large. In other words, a sizable FF system makes the economy profit-led. The political economy of income distribution gets complicated. Workers with FF or defined-contribution accounts face a dichotomy of their role in the class conflict since over their life-cycle they wear the coat of both the worker and of the wealth owner or rentier. In the first period workers earn a wage and based on (9) they should support pro-wage policies. Once a worker retires her income depends on the rate of profit and therefore she is interested in a higher profit rather than wage share. If a paygo system predominates the economy is expected to be wage-led. An increase in the wage share, all other things equal, increases social security benefits. Thus, those retirees who receive social security checks should have their interest sided with wage-earners.\(^6\)

\(^6\)To see this formally, we can write the level of pension as \( p = \rho wL = \rho^w = \rho w(1 + g) \).
the multi-dimensionality of the class conflict. It appears here that co-existing FF and paygo systems in a sense diffuses the classical conflict between capitalists and workers and, to some extent, it shifts it to the worker class.

4 Dynamics in an economy with a fully-funded, private system

4.1 Short-run dynamics

Recall that capacity utilization is a function of retirees’ share in capital which based on (9) is endogenous. To keep notation and analysis tractable, assume for now that investment demand is entirely exogenous and given by \( g = I/K \) and that income distribution is also given. To make dynamics visible we write \( \eta_t \) as:

\[
\eta = \frac{\tau \psi X_{t-1}}{K_t} = \frac{\tau \psi u_{t-1}}{1 + g}
\]

Capacity utilization has the following dynamics:

\[
u_t = \frac{g + s_w u_{t-1}/(1 + g)}{s_r[1 - s_w u_{t-1}/(1 + g)] + s_w}
\]

where \( s_w = \tau (1 - \pi) \) and \( s_r = s \pi \) are gross saving rates by workers and capitalists respectively, and are constant since income distribution and saving propensities are fixed. Equation (19) is a first-order difference equation for \( u_t \). At the steady-state when \( u_t = u_{t-1} = u^* \) two solutions are obtained by solving \( s_w s_r u^2 - u[s_r + g(s_w + s_r)] + g(1 + g) = 0 \):

\[
\begin{align*}
  u^*_1 &= \frac{g}{s_r} = \frac{g}{s \pi} \\
  u^*_2 &= \frac{1 + g}{s_w} = \frac{1 + g}{\tau \psi}
\end{align*}
\]

For those familiar with the Keynesian approach, the first equilibrium point is in fact the short-run static solution for \( u \) for an economy where only capitalists save. It is realistic to assume that \( u^*_2 > u^*_1 \). We can establish stability properties of these solutions by evaluating \( du_t/du_{t-1} \) at \( u^* \). The two conditions boil down to:

\[
\frac{du_t}{du_{t-1}} = \begin{cases} 
  \frac{s_w (1+g)}{s_r (1+g) + s_w}, & \text{at } u^* = \frac{g}{s_r} \\
  \frac{1}{1+g} + \frac{s_w}{s_w}, & \text{at } u^* = \frac{1+g}{s_w}
\end{cases}
\]

Stability requires \( du_t/du_{t-1} < 1 \). Statistics on developed economies suggest that the first solution is stable and that dynamics look as in figure 1.

Figure 1 here

Higher accumulation always stimulates the economy. A rise in the profit share
increases the average saving rate of capitalists and has the opposite effect on \( s_w \), the gross saving rate of workers. The short-run equilibrium solution \( u^*_1 \) declines while \( u^*_2 \) increases. To use a taxonomy promoted by [Dutt (1984), Taylor (1985)] or [Bhaduri and Marglin (1990)], \( u^*_1 \) is wage-led while \( u^*_2 \) is profit-led in the short-run. Our economy experiences a regime shift somewhere in between. As capacity utilization increases economic activity becomes profit-led. To stimulate this economy the solution is fairly simple. Either impose a tax on capitalists’ income share, or provide incentives such that the animal spirits captured somewhere in \( g \) are heightened. Notice also Pasinetti’s paradox. The saving behavior of capitalists drives the profit rate around \( u^* \), where the profit rate is solved to \( r = \pi u = g/s \). Unlike in Pasinetti’s full story, the economy adjusts to a change in \( s \) through a change in capacity utilization and therefore in employment, rather than through a change in income distribution.\footnote{Although the model examines an economy with excess capacity the results resemble those from models that assume an endogenous functional distribution of income [Kaldor (1955) and Pasinetti (1974)].} Retirees’ steady-state share of wealth at the stable equilibrium is equal to:

\[
\eta^* = \frac{\tau \psi g}{s \pi (1 + g)}
\]  

(23)

Capital accumulation (or output growth since \( g = \dot{X}/X \)) leads to an increase in employment and therefore to a rise in the number of retirees and their share in wealth \( \eta^* \). Interestingly, a FF pension system has no effect on the stable equilibrium solution \( u^*_1 \), but it does impact the economy on its transient paths. A rise in \( \tau \), which expands the FF system, rotates the curve depicting the \( u_t \) function counter-clockwise around the lower equilibrium point in Figure 1. The economy still converges to equilibrium, however at a slower speed as the rise in \( \tau \) increases the average saving rate for this demand-driven economy. If the economy happens to be below \( u^*_1 \) a higher contribution rate has an immediate negative effect on \( u_t \) and therefore on the profit rate \( r_t \). For current retirees this means a decline in their pension annuity, while current labor force have to put up with lower employment rates.

### 4.2 Accumulation in a FF economy

Although we have formally introduced dynamics in the model, the analysis so far is at best suitable for the medium-run. It is relevant to ask how pensions impact investment behavior and therefore capital accumulation in the long-run. This issue fits more broadly into the question of how the presence of retirees change the distribution of what is being produced in the economy in a given period and, therefore, the behavior of active factors of production i.e. labor and capital. For if a sizable part of profits takes the form of pension annuities then it is worth examining the ramifications for capital accumulation, innovation and social conflict in general. From the accounting viewpoint, retirees’ share in capital stock or wealth is
equivalent to a tax on profits.

\[ \pi^N = \frac{1}{X_t}(\Pi - qR_t r_t) = \frac{1}{X_t}(\Pi - \tau \psi X_{t-1} \pi u_t) = \pi(1 - \eta_t) \tag{24} \]

where \( \pi^N \) is capitalist’s net share of profits. In a most simple setting we formalize capital accumulation, which now is a state variable, to be a function of the net profit share and a target rate of profitability \( \bar{\pi} \): 

\[ \dot{g} = \lambda(\pi^N - \bar{\pi}) = \lambda \left[ \pi \left(1 - \frac{\tau \psi g}{s\pi(1 + g)} \right) - \bar{\pi} \right] \tag{25} \]

Stability of (25) is confirmed since \( d\eta/dg > 0 \) and therefore \( d\dot{g}/dg < 0 \). We can solve for \( g^* \) using the solution for \( \eta \) from (23):

\[ g^* = \frac{s(\pi - \bar{\pi})}{s(\bar{\pi} - \pi) + \tau \psi} \tag{26} \]

To make sense the numerator in (26) has to be positive and \( \pi > \bar{\pi} \). In other words the exogenously determined distribution between profits and wages as measured by the profit share is above the net profit share desired by capitalists. If this condition does not hold, capitalists and capital accumulation disappear from this economy. The long-run behavior of capital accumulation has some resemblance to the classical case. Saving affects investment, however capitalists and workers’ saving behaviors have opposite outcomes for \( g \). A higher profit share or saving rate by capitalists stimulates accumulation while workers’ saving, as captured by \( q = \tau \psi \), depresses it. The economic activity turns to be profit-led in the long run, a result warranted by the fact that \( du/dg > 0 \) and therefore \( du/d\pi > 0 \) given the above condition that the exogenously given profit share is larger than the desired net profit share, or \( \pi > \bar{\pi} \).

By specifying the dynamics of capital accumulation as in (25) we have implicitly gave the model a Goodwinian flavor: a social conflict exists over the distribution of earned profits between capitalists and retirees. If capitalists recognize that retirees’ share in future profits is a matter of their own choice of \( g \) then they can set on a level of capital accumulation that matches their desired level of profitability \( \bar{\pi} \). Retirees or workers for that matter do not own a weapon they can use to fight capitalists as is the case with the employment rate in Goodwin’s model (Goodwin (1967)). Thus, the economy reaches here a stable long-run equilibrium unlike circling around it.

The political economy of the FF system is indeed interesting. Once they reach retirement, workers switch allegiance to capitalists and in this way a rift is inflicted between generations of workers. If the political strength of retirees is significant and they team up with capitalists they can further weaken workers’ power without realizing that they are also working against future generations of retirees.\(^8\)

The model can be extended to include government spending and fiscal policy.\(^8\)

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\(^8\)The outcomes can take the form of legislative or institutional changes. The anti-labor and anti-union movement led by the legislative body in Wisconsin is a recent example of a conflict within labor. Workers from the private sector who lack unions and therefore certain securities and benefits have supported changes that weaken governmental unionized workers.
Intuitively, a rise in capacity utilization due to government spending causes an increase in employment and therefore in \( \eta \) with no effects on \( g \). Fiscal policy is said to be effective in the medium-run. The outcome of this type of policy intervention can be very different in the long-run if capitalists behave as in the exposition above. Fiscal policy turns to be ineffective should capitalists respond with a decline in capital accumulation \( g \) based on (25).

5 Dynamics in an economy with a paygo system

5.1 Short-run dynamics in a paygo economy

The variable of interest in a PAYGO economy is the ratio of retirees’ income to the wage bill as defined in (5). With the number of retirees determined by the economy in the previous period a dynamic \( \rho \) can be written as:

\[
\rho = \frac{\phi u_{t-1}}{u_t(1 + g)}
\] (27)

where \( \phi = p/w \). The solution for \( u_t \) is then solved from the short-run macro equilibrium condition.

\[
u_t = \frac{g + s_w \phi u_{t-1}/(1 + g)}{s_\pi + s_w}
\] (28)

where as before \( s_\pi \) and \( s_w \) are the average saving rates for capitalists and workers respectively. We assume that all pensions come from a paygo system and saving by workers are associated with other motives than old-age income. There is one fixed point for the capacity utilization given by:

\[
u^* = \frac{g}{s_\pi + s_w[1 - \rho^*/(1 + g)]}
\] (29)

For reasonable values of parameters, capital accumulation \( g \) has a positive effect on economic activity. The economy is clearly wage-led if capitalists have a higher saving propensity than workers. On the other hand, retirees always stimulate a demand-constrained economy through \( \phi \), however the magnitude of the change in economic activity depends on workers’ saving rate, \( s_w \). In this sense Japanese retirees may very well have a greater impact on their economy compared to retirees in the US since saving rates of wage-earners are considerably higher in the former. Notice that at equilibrium the tax rate is \( \rho^* = \phi/(1 + g) \) and it is a function of the pension-wage ratio and the growth rate \( g \). Retirees grow at the same rate as workers which is the growth rate of the economy. To see the effects of changes in the social security tax on the economy we can rewrite the solution for \( u \) as:

\[
u^* = \frac{g}{s_\pi + s_w[1 - \rho^*]}
\] (30)

\[^9\text{(27) follows from } \rho = \phi R/L = \phi X_{t-1}/X_t = \phi X_{t-1}/(K_{t-1}(1 + g)X_t/K_t)\]
The model’s stability requires the partial derivative of $u_t$ with respect to $u_{t-1}$ evaluated at $u^*$ to be less than unity.

\[ \frac{du_t}{du_{t-1}} = \frac{\phi s_w}{(s_\pi + s_w)(1+g)} \]

Stability is met as long as $\phi$ is not very large. In fact, in most cases, $\phi < 1$. The economy’s dynamics appear in figure 2. An increase in either capital growth rate or pension relative to wage shifts $u_t = f(u_{t-1})$ upwards towards a higher equilibrium point.

**Figure 2 here**

Let’s briefly look now at the dynamics when there is class conflict of the sort described in section 3.4 and workers attempt to share costs imposed by the social security system through the tax $\rho$ with the capitalists. To keep notation simple we turn off workers’ saving behavior. The inverse relation between $\rho$ and $\pi$ can be formalized as $\pi = \rho^\sigma = [\phi u_{t-1}/u_t(1+g)]^\sigma$ where $\sigma < 0$. This simple formulation means that a rise in $\rho$ brings down $\pi$ at the rate of $\sigma$. A rise in the current period capacity utilization restrains $\rho$ and therefore eases the distributive conflict between workers and capitalists. The first-order difference equation for $u$ is:

\[ u_t = \left( \frac{g}{s_\pi} \right)^{1/(1-\sigma)} \left( \frac{1+g}{\phi u_{t-1}} \right)^{\sigma/(1-\sigma)} \]

where $s_\pi$ is now the marginal propensity to save out of profits. The equilibrium solution for (32) is $u^* = (g/s_\pi)(1/(1+g))/\phi]^{\sigma}$ and the economy is wage-led. The equilibrium tax rate is as before given by $\rho = \phi/(1+g)$. Steady-state capacity utilization is now $u^* = (g/s_\pi)(1/\rho){\sigma}$. A higher social security stimulates capacity utilization given that $\sigma < 0$. The transfer of income from capitalists, the saving class, to retirees ensures this income. At the same time profitability in this economy as measured in terms of the profit rate is unaffected by the existence of a social security system. Similar to the FF economy, the profit rate responds only to the saving and investment behavior of capitalists, $r = \pi^* u^* = g/s_\pi$. The stability of the model requires that evaluated at equilibrium $du_t/du_{t-1} < 1$ which boils down to:

\[ \frac{du_t}{du_{t-1}} = \frac{\sigma}{\sigma - 1} \]

It is straightforward that with $\sigma < 0$ the system is indeed stable. These dynamics are similar to those described in figure 2. The result has to do with two features of this economy. First, demand is pension-led, and second, the pension bill in the national income declines with better economic activity. Unlike the typical forced saving story (see Barbosa-Filho and Taylor (2007)) where a rise in capacity utilization leads to a decline in the wage share, here, better current economic performance makes it easier to sustain current retirees. In order words, $\rho$ is kept under control.
Starting from an initial $u_0$ situated at the left of equilibrium, a larger current capacity utilization translates into an increase in the number of retirees next period and therefore a higher aggregate demand. This rise in demand is the outcome of a successful bargaining by workers who force capitalists to support a part of new retirees’ consumption.

5.2 Policy in a paygo economy

I introduce now dynamics on capital accumulation and an active fiscal policy. The model concerns an economy with a paygo system, low saving rates and an endogenous social security tax. The government or the policy maker recognizes three important aspects related to the economy and the pension system. First, it understands that better economic performance eases the taxation burden on employees at the steady-state since $\rho^* = \phi/(1 + g)$, as well as along transient paths as a result of $\rho_t = \phi u_{t-1}/u_t(1 + g)$. Second, it is aware that high taxation of workers may be disruptive and promote conflict between workers and capitalists over sharing the burden of pensions. Third, it knows there is unused capacity and therefore insufficient demand in the economy. The policy rule for government spending is set to:

$$G_t = \mu(\rho_t - \tilde{\rho})X_t$$

(34)

where $\mu$ is a reaction coefficient and $\tilde{\rho}$ is the target tax rate. Current tax rate remains endogenous and given by (27). We assume that workers do not save and contributions by workers are entirely used by retirees for their current consumption. In an economy with no government intervention and no class conflict the assumption of zero savings by workers means no impact of a paygo scheme on effective demand and therefore on the economy. However, pension benefits or the tax rate would have to change if there is a change in the growth rate of the economy. The macro balance in an economy with an active fiscal policy is:

$$X_t = (1 - s)\pi X_t + (1 - \pi)X_t + \mu(\rho_t - \tilde{\rho})X_t + I$$

(35)

Dividing (35) by $K$ we obtain the following first-order difference equation for capacity utilization:

$$u_t = \frac{g + \mu \phi u_{t-1}}{s\pi + \mu \tilde{\rho}}$$

(36)

At equilibrium capacity utilization is:

$$u^* = \frac{g}{s\pi + \mu(\tilde{\rho} - \rho^*)}$$

(37)

where as before $\rho^* = \phi/(1 + g)$ is the equilibrium tax rate. A higher policy target $\tilde{\rho}$ means lower government spending and therefore lower capacity utilization at the steady-state. A change in $\tilde{\rho}$ does not affect the prevailing steady-state tax rate since
both $\phi$ and $g$ are exogenous. However, over the transition to a new equilibrium, the tax rate is increasing in the immediate period, since a decline in capacity utilization and output lowers current employment but has no effect on the current cohort of retirees, their number being set by previous economic performance. In other words, a contractionary fiscal policy can be damaging for the solvency of a paygo scheme. How about a change in capital accumulation or in benefit-wage ratio? A lower investment demand is equivalent with a lower aggregate demand, and, at the same time, a higher tax rate $\rho$. Fiscal policy kicks in and a rise in government’s spending alleviates to some degree, the effect on economic activity, $u$, but without having any impact on the steady-state tax rate $\rho^*$. Once again, the intervention only succeeds in easing the tax burden over transient paths. 

In the long-run the ratio of debt to capital stock becomes the relevant state variable. Assuming that there are no taxes, and working with a differential rather than a difference equation, the growth rate in debt is given by:

$$\frac{\dot{D}}{D} = \mu(\rho^* - \hat{\rho}) X D + i$$

(38)

where $i$ is the interest rate government pays on its debt. Taking the time differential of the debt-capital ratio $\delta = D/K$ we get an expression for debt dynamics:

$$\dot{\delta} = \mu(\rho^* - \hat{\rho}) u + (i - g) \delta$$

(39)

The differential equation (39) is stable as long as the growth rate of the economy is above the prevailing interest rate. Assuming this condition holds a steady-state solution for $\delta$ is:

$$\delta^* = \frac{\mu(\rho^* - \hat{\rho}) g}{(g - i)(s\pi + \mu(\hat{\rho} - \rho^*))}$$

(40)

The first thing to notice in (40) is that government may never reach its policy target $\hat{\rho}$. This result further implies a positive debt-capital ratio at the steady-state. The government does not control capital accumulation $g$ and hence can not pick a value of $g$ that ensures $\rho^* = \hat{\rho}$. The best it can do for now, is to have an impact on economic activity during transitions to new steady-states. To borrow an analogy from Kalecki (1943) policies that target full employment may require continuous budget deficits. A more tolerant attitude towards higher taxes $\hat{\rho}$ reduces government’s debt, but as we have seen above, this happens at the expense of economic activity as measured by $u$. The analysis and consequently these results are obtained using a rather extreme version of government’s finances since we do not allow government to collect any taxes. Should we change this assumption, the results become favorable towards an active fiscal stance (Jong-Il and Dutt (1996)).

At the steady-state a higher $\rho^*$ causes a larger debt to capital ratio. Even if fiscal policy restrains the tax burden over the medium-run, in the long run it returns to the steady-state as given by $\phi/(1 + g)$. These results may cast doubt on the efficacy
of such intervention as long as capital accumulation and productivity are exogenous. A different story prevails if investment responds to a higher capacity utilization. $g$ becomes a state variable with dynamics given by:

$$\dot{g} = g[f(u^*) - g] = g\left(\frac{g}{s\pi + \mu(\bar{\rho} - \rho^*)}\right) - g \tag{41}$$

The economy is described by a system of differential equations, (39) and (41). Debt-accumulation dynamics are stable as long as $g > i$ and $d\dot{g}/g < 0$. The latter is analogous to the Keynesian stability condition that investment demand responds less strongly than saving dynamics to economic activity. The Jacobian for our system is:

$$J = \begin{bmatrix} \frac{g}{s\pi + \mu(\bar{\rho} - \rho^*)} & 0 \\ h'(g) - \delta & i - g \end{bmatrix}$$

where $h(g)$ is the first term on the r.h.s of the differential equation for debt dynamics (39). If the two local stability conditions hold and capital accumulation negatively impacts debt we get a phase diagram as in figure 4.

**Figure 4 here**

The policy target $\bar{\rho}$ can now have an impact. A decrease in $\bar{\rho}$ is equivalent with a more active fiscal policy and therefore increased government’s spending. Investment responds to a higher capacity utilization and the nullcline $\dot{g} = 0$ shifts upwards. A decline in the prevailing steady-state tax rate $\rho^*$ follows which slows down fiscal intervention and therefore eases deficit pressures. Depending on the response of debt to a higher growth rate, a more active fiscal policy and a lower tax burdens on the economy, the new steady-state $\delta^*$ may be lower or higher. As drawn in figure 4, the outcome, a decline in the new steady-state debt-capital ration, is beneficial for the government’s fiscal position, while the tax burden of the paygo system is kept under control.

6 Conclusions

The following table summarizes main results with respect to the type of class conflict i.e. primary distribution (PD) and secondary distribution (SD), qualitative features of short-run equilibrium and steady-states solutions, implications for the economy over transient paths and policy for fully-funded and paygo pension schemes.

**Table 1 here**

Class conflict is more nuanced in the FF economy. It departs from the classical case with clearly separated economic classes. Labor’s interests are divided between its role as a worker and as a retiree. Capitalists engage in the distributive conflict with both workers and retirees but it is difficult based on the models in this paper to pinpoint how dynamics are likely to unfold. One would need to endogenize income
distribution by adding constructs such as a wage curve, and ideally a mechanism for productivity growth. What I hope to have suggested more clearly is that the benefits from a FF scheme in the long-run are far from certain and subject to how accumulation reacts to a secondary distribution of income. In addition fiscal policy finds itself restrained, and I think, it is being stripped of the channels through which it can diffuse some of the conflict over income distribution. In comparison, fiscal policy can help a paygo economy both with regard to growth and class conflict.
A Figures

Figure 1: Dynamics in an economy with a fully-funded pension system

Figure 2: Dynamics in an economy with a paygo pension system and no distributive conflict
Figure 3: Debt and accumulation dynamics in an economy with a paygo pension system and active fiscal policy
### Tables

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Table 1: A contrast of fully-funded and paygo pension schemes
References


