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Social Security Tax and Endogenous Technical Change in an Economy with an Aging Population

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Abstract

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Keywords: Population aging; Social security tax; Endogenous technical change
JEL Classification: E62; E24; O30

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1 Introduction

Social conflict surrounding the distribution of income in an economy undergoing significant population aging can become multi-dimensional. The factor of production designated to transfer part of its income to the dependent population will seek to pass on or at least share these costs with the other active factors of production. The secondary distribution of income can therefore have significant effects on the primary distribution of income and therefore on relations of production in the economy.

This paper focuses on an economy with an unfunded social security system. I present here a macroeconomic model that combines a Goodwin (1967) model of cyclical growth and accumulation with induced technological change and a wage curve which I extend to account for the effects of a social security tax. The model is built on previous work by Shah and Desai (1981) and more recently Foley (2003). In this framework output growth is set from the supply side by available savings.

In addition to giving income distribution a prime place in the analysis I also tackle the question of how technical change may evolve in an economy with rising old-age dependency

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rates. Should we expect firms to intensify their efforts to promote labor-augmenting technical change as labor supply declines? Or will firms watch passively and do nothing more than keep the labor to capital ratio at some steady level? In this paper the answers to these questions depend on the type of social conflict about the distribution of income among various economic classes. The theory of induced technical change was formalized first in a complete framework by Kennedy (1964). It justifies the bias of technical change based on the shares of factor cost in total costs. Applied to our topic, pressures from labor costs due to higher dependency rates may induce firms to opt for labor-saving technical change.

The government plays a very limited role. Government's only responsibility is to set the social security policy and maintain a balance between taxes collected from workers and capitalists and transfers to the retired population. These choices, and in particular the balanced budget, place significant constraints on macroeconomic policies. Nevertheless, there are compelling reasons for modeling government behavior in this manner. First, restricting the presence of government in the model allows us to highlight the dynamics and social conflict among current factors of productions and the retired population. Second, these choices do not preclude us from suggesting how the results could change should the government have more to say with respect to fiscal policies or be allowed to run a budget surplus or deficit.

The paper explores four models which are differentiated on the basis of the social security tax and the process of technical change. The first model, which takes both social security tax and technical progress as exogenous, exhibits the same dynamics as the original Goodwin model of cyclical growth. Fiscal policy does not alter the *net* distribution of income between profits and wages, however, it does affect the rate of employment. From retirees' perspective an optimal social security tax rate exists which maximizes the individual pension level.[1] In the second model technical change is endogenous, similarly to the models by Shah and Desai (1981) and Foley (2003). The steady-state employment rate continues to respond negatively to a hike in the social security tax. An important difference with the previous case is that for a period of time labor productivity grows faster during the transition to the new steady-state due to induced technical change. The endogeneity of technological change also adds new powers to redistributive fiscal policy. The next two models take the social security tax as endogenous. When technical progress is exogenous the system exhibits two fixed points. The endogeneity of the social security tax and plausible initial conditions ensure that the economy reaches the steady-state with a higher rate of employment. The last model highlights the fact that endogenous technical change can stabilize the old-age dependency rate at least for a period long enough for the economy to reach the plateau of its demographic change.

My approach to modeling the macroeconomic implications of population aging differs from what is usually found in the literature. The typical overlapping-generations model studies the viability of different pension systems with a focus on the effects on saving behavior by economic classes and therefore growth and how these may respond to changes within or between various systems of old-age income provision.[2] Factor payments in the neoclassical OLG models with exogenous technology are determined in the process of (efficient) allocation of resources and are paid according to their marginal productivities. When the full employment assumption applies wages have to be at a level such that savings from wages are just enough to provide for the capital stock necessary to employ the entire labor force (Foley and

[1]As discussed later on the social security tax is optimal from the point of view of retirees who want to maximize their income. It may however not be optimal for the economy overall and in particular for labor who will see lower levels of employment.

[2]See seminal papers by Feldstein (1974), Barro (1974) and Kotlikoff (1979), and versions of the model with endogenous growth by Saint-Paul (1992), Zhang (1995) and Zhang and Zhang (1998).

Michl (1999)). In our framework labor remuneration is set by a wage curve that captures class tensions arising from the secondary distribution of income. As a result unemployment is possible.

But there are also similarities with results found in some of the mainstream literature. Redistribution of income from capital to labor does not always hurt growth (Bertola (1996), Bertola et al. (2005)). OLG models with endogenous growth also pay attention to the effects that technology has on income distribution and therefore growth. Zuleta (2008) presents a model where innovation is contingent on the relative factor abundance. In this case the developed economies which are capital abundant are likely to see labor-saving technology and higher long-run growth. A similar mechanism is introduced here, even though from the perspective of the social conflict embedded in the wage curve. From the non-orthodox perspective Barba (2006) uses a social security tax in wage bargaining but closes the model from the demand side by assuming excess capacity utilization. Changes in income distribution are then possible if the central bank controls inflation by setting the long term real interest rate. Consequently, income redistribution has a positive effect on accumulation and output in the short run if workers' and retirees' propensity to consume is larger than that of capitalists'.

Aside from this introduction, the current paper starts with an examination of national accounting for an economy with significant population aging. The next section describes the main building blocks of the model. Sections 4 and 5 develop the four models based on different scenarios concerning the nature of the social security tax and the mechanism of technical change. The paper concludes with few simulation results and policy recommendations.

2 Accounting for an economy with significant transfers

Many macro problems can be illuminated by a first look at national income accounting. In this section I discuss the main components of national income and its allocation at different stages. Primary distribution of national income X apportions earnings of factors of production, capital K and labor L, according to:

$$X = W + \Pi \tag{1}$$

where W equals the wage bill and Π represents total profits. Written in primary income shares (1) becomes:

$$1 = \psi + \pi \tag{2}$$

where $\psi = wL/X$ is the wage share and $\pi = rK/X$ is the profit share. Dubbed *the two souls problem* (2) offers a way to introduce class struggle and income distribution in the analysis of economic dynamics [3]. The constraint that (2) places on the allocation of income is rather obvious: an increase in the wage share can take place only at the expense of the profit share in national income. In this paper I extend the two souls problem to include the retired population as an important player in the determination of income distribution. To do this we need to look at the secondary distribution of income.

National income is redistributed through transfers to main economic classes or institutions – households, businesses and government. In this paper pensions come entirely from workers' income through a social security tax ρ applied on wages. If government's only role is to set

[3] Flaschel (2009) borrows the analogy of the two souls from Goethe's Faust. Taylor (2011) discusses at great length the implications of an independent theory of income distribution for the analysis of macroeconomic dynamics.

the social security tax and to keep a balanced budget we can re-write the national income relation (1) as:

$$X = W(1-\rho) + pR + \Pi \tag{3}$$

where p is the level of pensions and R is the number of retirees. Dividing (3) by L we get:

$$\epsilon_L = w(1-\rho) + pd + rk \tag{4}$$

where $d = R/L$ is the demographic dependency rate, ϵ_L is labor productivity, r is the profit rate and k is the capital-labor ratio. Relation (4) is the income-schedule for this economy and describes how current income is divided among economic classes. Direction and magnitude of class conflict depend on the nature of the social security tax. I assume two different scenarios for ρ which, as explained in more detail below, have very different implications for class conflict. In the first scenario ρ is set by policy at a level $\bar{\rho}$. The retiree's individual income follows from:

$$p = \frac{\bar{\rho} w L}{R} \tag{5}$$

If the retired population is politically weak and therefore unable to lobby successfully for an increase in $\bar{\rho}$, the individual pension level declines as the number of retirees, R, increases. In this situation class conflict is suppressed beyond the two souls problem captured by (2). On the other hand, if policy responds to retirees' demands for a rise in the social security tax, the after-tax wage share or workers' disposable income declines. It can be expected that workers would bargain for higher wages. If they are successful, the primary distribution of income changes and as a result production relations change. Hence, the two souls problem is said to be extended to the three souls problem by accounting explicitly for the role retirees play in the determination of income distribution.[4]

In the second scenario ρ follows endogenously from the accounting equation:

$$\rho = \frac{\bar{p} R}{\omega L} \tag{6}$$

where $\bar{p}$ is a fixed level of individual pension. As the number of retirees increases the social security tax has to rise unless the wage level goes up. Looking at (6) one may become less hopeful about the prospects for an economy with an aging population. Class conflict seems bound to become a staple for the economy as wage bargaining is now being internalized on a continuous basis. What I hope to show is that depending on the particular process of technical change, class conflict has the potential to set the economy on a dynamic path that allows it to successfully accommodate an aging population.

Output in the national accounts can be described also from the demand side. For a closed economy we can write total output as the sum of consumption by workers and retirees and investment:

$$X = C^w + C^r + I \tag{7}$$

which divided by L becomes:

$$\epsilon_L = c^w + c^r d + g_k k \tag{8}$$

where c^w and c^r are consumption levels per worker and per retiree respectively and g_k is the growth rate of capital stock. The consumption-growth schedule described by (8) shows how

[4] In order to formalize the bargaining process, social conflict appears now to be clearly divided between three economic classes: workers-capitalists and workers-retirees. But at the same time, in a PAYGO system, workers may recognize the intertemporal similarity between their own class and that of retirees. They can therefore be perceived as one class and bargain to share the costs of current retirement with capitalists.

current output is being distributed between social consumption, conducted in this case by workers and retirees, and investment (Foley and Michl (1999)). In a supply driven model (8) underlines the trade-off between current consumption and future output, where the latter is a function of the rate of investment.

In both (4) – the wage-profit relationship – and(8) – the consumption-growth relationship – the constraints on the distribution of income or consumption versus investment are relaxed with higher labor productivity growth. Using $rk = \pi\epsilon_L$ we can rewrite (4) as:

$$\epsilon_L(1-\pi) = w(1-\rho) + pd \tag{9}$$

In (9) an increase in labor productivity allows the economy to sustain a higher dependency rate without undergoing changes in the distribution of income. A constant ψ requires wages to grow at a rate dictated by the labor productivity. As a result the wage bill goes up which makes it possible to provide for a larger share of elderly without a rise in the tax rate or a reduction in the pension level.

3 The model

A useful representation of the macrodynamics of social conflict accompanying income distribution has been put forth by Richard Goodwin in a 1967 model of cyclical growth. At the core of this model is the two souls problem described above. It is formalized by giving workers the employment rate, l, as a weapon in bargaining for higher wages, and to capitalists the investment decision represented by, g_k, as the means to determine the growth of employment (Shah and Desai (1981)). In this paper I extend Goodwin's model of growth by introducing the third soul, retirees and their pensions, into the income distribution problem. A crucial feature of the Goodwin model is that unemployment is compatible with a saving-driven investment.[5]

3.1 Production

For simplicity, assume a single-good economy characterized by a Leontief technology embedded in a production function that uses labor L and capital K according to:

$$f(K, L) = min[\bar{L}, \bar{K}] \tag{10}$$

where $\bar{L} = \epsilon_L L$ and $\bar{K} = \epsilon_K K$. With a Leontief technology, output is obtained based on $X = \bar{L} = \bar{K}$. It follows that the demand for labor can be derived as:

$$L^d = \frac{X}{\epsilon_L} = \frac{\bar{K}}{\epsilon_L} = \frac{K\epsilon_K}{\epsilon_L} \tag{11}$$

Labor demand in (11) increases when either the available stock of capital or capital productivity expand. On the other hand, more productive labor implies less need for additional labor input. Taking the total differential of (11) labor demand expands according to:

$$\frac{\dot{L}^d}{L^d} = \hat{K} + \hat{\epsilon}_K - \hat{\epsilon_L} \tag{12}$$

[5]The saving-led investment highlighted during the capital controversy debate is a contentious assumption for those working in the heterodox tradition. An interesting discussion between Michl (2006) and Cesaratto (2006) highlights some of the difficulties and methodological tradeoffs we face in addressing crucial political economy questions in formal economic models. Despite adopting Say's Law Goodwin's model avoids making additional assumptions that would deliver full employment and therefore would limit much of the policy discussion and would make its conclusions quite unrealistic.

where a "'hat"' stands for a rate of change. Workers consume their entire disposable income and therefore capital accumulation is made possible by available saving conducted by capitalists only who instead are assumed to save all their profits:[6]

$$\frac{\dot{K}(t)}{K(t)} = g_k = r = \pi\epsilon_K = (1-\psi)\epsilon_K \tag{13}$$

In this setup a shift in income towards wages has adverse effects on the rate of investment and therefore on the growth rate of the demand for labor:

$$\frac{\dot{L}^d}{L^d} = (1-\psi)\epsilon_K + \hat{\epsilon}_K - \epsilon_L \tag{14}$$

3.2 Labor market

Building parsimonious economic models is impossible without making restrictive and often controversial assumptions. These are most often related to how labor markets work. There are three main variables that characterize the labor market: labor supply, L^s; labor demand, L^d; and the real wage, w. In this paper, labor demand is determined according to (14) while labor supply is exogenous and grows at a rate, n:

$$\frac{\dot{L}^s}{L^s} = n \tag{15}$$

From (12), (13) and (15) we obtain the growth rate of the employment rate, l as:

$$\frac{\dot{l}}{l} = (1-\psi)\epsilon_K + \hat{\epsilon}_K - \hat{\epsilon}_L - n \tag{16}$$

Following Goodwin (1967), wages respond to how tight labor markets are based on a real wage Phillips curve. An increase in the employment rate provides workers with better chances for successful wage bargaining for a higher *growth rate* of the real wage. In this paper I extend the real wage Phillips curve to account for changes due to the secondary distribution of income. A plethora of research on the effects of tax regimes on wage setting and unemployment motivates the introduction of social security tax in the wage bargaining.[7] Specifically, an increase in the social security tax following a rise in the share of retirees drives workers to pass on some of these costs to firms according to:

$$\frac{\dot{w}}{w} = \delta_1 l + \delta_2 \rho \tag{17}$$

[6] The assumption of zero capitalist consumption in the presentation of the model is made in order to simplify the notation. Simulation results discuss later on (see Appendix C) are based on a discount rate of 0.1. The use of a non-zero consumption by capitalists does not modify the qualitative features of the model; only the equilibrium solutions for the state variable. Foley (2003) for example shows that for the typical Goodwin model a negative shock to the saving rate reduces the steady-state wage share.

[7] In Lockwood and Manning (1993), after-tax wages are determined based on a Nash bargaining process between unions and firms where firms choose the level of employment. One interesting conclusion is that the progressivity of the tax system reduces wage pressures in the economy. This would suggest in our case and specifically for the US economy that applying the social security tax on wages above the current maximum of USD 106,800 may ease the intergenerational conflict. Other literature that extends the wage bargaining process to take into account taxes can be found in Layard (1982) and Koskela and Schb (1999) among others.

where $\delta_1 > 0$ and $\delta_2 > 0$. The law of motion for the wage share follows from (17):

$$\frac{\dot{\psi}}{\psi} = \delta_1 l + \delta_2 \rho - \hat{\epsilon}_L \tag{18}$$

Equations (16) and (18) are the core relations of our model and we can use them to explore macroeconomic dynamics arising in an economy with significant population aging.

3.3 Factor productivities and technical change

The last building block of the model concerns the process of technical change and determination of factor productivities. I explore the following two cases. First, I analyze the dynamics of the model assuming exogenous technological change. Thus, both the growth rates and levels of factor productivity are exogenous. For a steady-state to exist technological change must be Harrod-neutral or purely labor-augmenting:

$$\hat{\epsilon}_L = \frac{\dot{\epsilon}_L}{\epsilon_L} = \gamma \text{ and } \hat{\epsilon}_K = \frac{\dot{\epsilon}_K}{\epsilon_K} = \chi = 0 \tag{19}$$

From now on I shall be using γ and χ to denote the growth rates of labor and capital productivity respectively.

Next, I extend the model to investigate an economy with endogenous technical change. We can think of firms as maximizing profits by reducing the use of the most expensive inputs through an increase in the productivity of these inputs. Technical change therefore becomes biased towards the input with the highest share of costs in total cost of output. The literature (Dumenil and Levy (1995), Dumenil and Levy (2003), Kennedy (1964), Foley (2003)) suggests that the growth of productivities of factors of production are functions of the shares of factor costs according to:

$$\hat{\epsilon}_L = \gamma[\psi] \tag{20}$$
$$\hat{\epsilon}_K = \chi[1 - \psi] \tag{21}$$

The constraint this problem places on the economy or the firm is that fewer savings are made on the input which is not being targeted by technical change. Following Foley and Michl (1999) let the technical progress function be given by a set of techniques which determine the growth rates of labor and capital productivity such that:

$$\gamma = f(\chi) \tag{22}$$

where the constraint on the firm translates as $f'(\chi) < 0$. The firm maximizes the cost savings function:

$$max = (1 - \pi)\gamma + \pi\chi \tag{23}$$
$$\text{subject to } \gamma = f(\chi) \tag{24}$$

Applying the first order condition to (23) the solution for χ follows from:

$$f'(\chi) = -\frac{(1 - \psi)}{\psi} \tag{25}$$

where f' is the slope of the technical progress function which increases with a higher ψ and the production shifts towards labor-augmenting and capital-using techniques.

3.4 Population aging in Goodwin's model

There are two aspects of the proposed model which require further discussion. First, Goodwin's model is a small model and its intention is 'not so much description as it is basic training for the intuition' Solow (1990). Since its publication in 1967 the paper has led many to test empirically for the existence of the predicted clockwise cycles between the wage and the employment shares. The cycles exist and they are clockwise but they do not have the periodicity of the usual business cycles and they are not the closed orbit cycles produced by a Lotka-Volterra set of equations. Rather, the data seems to suggest that the conflict (ψ, l) follows a cyclical pattern which shifts across time. The question is what are those factors that produce the observed changes in the position of the cycle? Related to the topic of this paper it is relevant to ask whether population aging and by extension the conflict surrounding the distribution of income act as forces that could produce and explain such shifts in the (ψ, l) dynamics and therefore in relations of production.

Secondly, population aging is a transitory phenomenon. We are therefore urged to think about two distinct types of issues: issues that are relevant at the steady-state and issues that concern transient paths. The steady-state analysis is conducted in the context of no population aging since demographic changes are expected to reach a plateau. More pronounced aging of population during the demographic transition produces a higher *level* of old-dependency rate at the steady-state. Consequently, the steady-state tax rate is higher and has direct implications for the steady-state solutions of other endogenous variables. Transient paths on the other hand are directly impacted by the process of population aging. The relevant policy question we should ask is how the economy will manage the transition to the new demographic steady-state. Is the mechanism of endogenous technical change a potential solution for a temporary problem? The difficulty we face is that a formal investigation of transient paths is mathematically cumbersome. Instead, we can make use of numerical simulations to understand complicated dynamics. Finally, there is also the question of the length of the demographic transition. Using forecasts on fertility rates and life expectancy from the United Nations Population Databases[8] Attanasio et al. (2007) predict a rise in old-age dependency rates throughout this century but with a significant slowdown after 2050. Since we are talking about considerable demographic changes at the least of almost a half a century the urgency of trying to understand dynamics over the transition period is even more apparent.

4 Exogenous social security tax

In the first set of models fiscal policy sets the social security tax to $\bar{\rho}$. According to the analysis in section (2), the pension level, p, becomes endogenous.

4.1 Exogenous technical change and exogenous taxes - Model A

When technical change is exogenous the dynamics in this economy depend only on the interaction between the employment rate and the wage share according to the Goodwin model of

[8] World Population Prospects: The 2002 Revision Population Database (2003), and World Population in 2300.

cyclical growth:

$$\frac{\dot{\psi}}{\psi} = \delta_1 l + \delta_2 \rho - \gamma \tag{26}$$

$$\frac{\dot{l}}{l} = (1-\psi)\epsilon_K + \chi - \gamma - n \tag{27}$$

With Harrod-neutral technical change, the system solves at the non-trivial fixed points, $\psi^* = 1 - (\gamma + n)/\epsilon_K$ and $l^* = (\gamma - \delta_2\rho)/\delta_1$. The steady-state profit rate can also be calculated and is equal to $r^* = \gamma + n$. Introducing an exogenous social security tax does not alter the qualitative features of the Goodwin model. An attempt by workers to share pension costs with capitalists has no effect on the distribution of income or the profit rate. Instead it leads to a lower rate of employment at the steady-state around which the economy oscillates as visually described in figure 1.[9]

[**Figure 1 here**: Exogenous social security tax and exogenous productivity]

This finding suggests that social conflict in an economy as the one described above could hinder efforts to economically sustain an aging population. Workers are expected to suffer economically as fewer jobs become available (at the steady-state).

We can use the model to gain insights with respect to the net gains for retirees. Let us assume that the economy is at the steady-state as given by l^* and ψ^*. The laws of motion for the wage level, the labor force and the retired population are provided by $w(t) = w_0 e^{\gamma t}$, $N(t) = N_0 e^{nt}$ and $R(t) = R_0 e^{n_r t}$ respectively. After grouping relevant terms the sequence for the pension level is:

$$p(t) = \frac{\rho w^* L^*}{R} = \frac{N_0 w_0}{R_0 \delta_1} e^{(\gamma + n - n_r)t}(\rho\gamma - \delta_2 \rho^2) \tag{28}$$

Taking the partial derivative of (28) in respect to ρ we get:

$$\partial p(t)/\partial \rho = \Theta(\gamma - 2\delta_2\rho) \tag{29}$$

where $\Theta = \frac{N_0 w_0}{R_0 \delta_1} e^{(\gamma+n-n_r)t} > 0$. In (29) the pension level increases with a rise in the social security tax up to the point where $\rho = \gamma/2\delta_2$. If the social security tax passes this critical value the decline in employment becomes large enough to trigger a sizable decline in the wage bill and therefore a fall in the level of pensions. In other words, in this most simple specification of the model, additional unemployment appears necessary to neutralize the conflict between retirees and workers. We also notice that the capacity for fiscal redistribution depends on the rate of labor productivity growth. A higher γ provides more space for policy makers to elevate ρ without hurting retirees' income but rather expanding it. Additionally, faster expansion of labor productivity benefits the steady-state rate of employment. Once again, these results underline the important role of labor productivity growth for an economy with an aging population.

As an alternative to the social security tax on wages one could argue in favor of a tax on profits. Julius (2005) shows that fiscal policy aimed to promote burden sharing through taxation of profits is futile as both the profit rate and the *net* distribution of income remain

[9] It would be more accurate to say that the cycle will shift to a lower interval for the employment rate as shown in Figure 1.

unchanged at the steady-state. A tax on profits τ means that investment is now made possible by after-tax profits. The law of motion for the employment rate changes to $\dot{l}/l = \pi(1-\tau)\epsilon_K - \gamma - n$ where $\pi(1-\tau) = \pi_N$, the net profit share, remains unchanged since $\pi_N = (n+\gamma)/\epsilon_K$ continues to hold. Before-tax income distribution is now characterized by $\pi^* = (n+\gamma)/[\epsilon_K(1-\tau)]$ and $\psi^* = 1-(n+\gamma)/[\epsilon_K(1-\tau)]$ respectively. In other words, retirees' income supplemented through a tax on profits is in fact a substitute for workers' wage.[10] There is however a crucial difference between taxing profits to sponsor retirees or taxing wages through ρ. In the former case the employment rate remains unchanged at the steady-state whereas a rise in the social security tax always results in a lower l^* and therefore lower growth. A similar qualification of fiscal policy effects due to a wage versus a profit or a capital tax is reached by Michl and Foley (2004) and Michl (2009). They show that prefunding a social security reserve system from a tax on capital increases the level of capital, output and employment.[11]

4.2 Induced technical change and exogenous taxes - Model B

I now extend the model to include endogenous technical change. The workings of the economy are described by a dynamical system which is very similar to the ones developed by Foley (2003) and Shah and Desai (1981):

$$\frac{\dot{\psi}}{\psi} = \delta_1 l + \delta_2 \rho - \gamma[\psi] \tag{30}$$

$$\frac{\dot{l}}{l} = (1-\psi)\epsilon_K + \chi[1-\psi] - \gamma[\psi] - n \tag{31}$$

$$\frac{\dot{\epsilon}_K}{\epsilon_K} = \chi[1-\psi] \tag{32}$$

The distribution of income at the steady-state follows from the condition $\hat{\epsilon}_K = \chi[1-\psi] = 0$ and is given by $\psi^* = \chi^{-1}(0)$. As pointed out by Foley (2003) the steady-state wage share has to be at a level that brings the growth rate of capital productivity to zero otherwise the system does not have an equilibrium. The fixed points for the other two state-variables, ϵ_K and l, are obtained from $\hat{l} = 0$ and $\hat{\psi} = 0$ respectively and are $\epsilon_K^* = (n+\gamma[\psi^*])/(1-\psi^*)$ and $l^* = (\gamma[\psi^*] - \delta_2\rho)/\delta_1$. Applying the Routh-Hurwitz stability criteria (discussed in an appendix) we can show that the system is likely to be stable. Thus, the Goodwin model with endogenous technical change converges towards a locally stable equilibrium given by $l^*, \psi^*, \epsilon_K^*$. Shah and Desai (1981) point out that these particular dynamics result from a change in the balance of power which has tilted in favor of capitalists. They can now choose not only the level of investment but also the technique of production that returns the economy to its steady-state.

The equilibrium rate of employment continues to respond negatively to a higher social security tax. Similar to the model with exogenous technical change, the increase in the wage share has a direct impact on the law of motion of l, equation (31), through capital accumulation. At the same time, successful bargaining by workers to share pension costs with

[10] In terms of the national income accounting we have $\pi = \pi_N + \phi$ where ϕ is the share of income going to retirees. Before and after-tax wage shares are the same, $\psi = \psi_N$ (relative to the previous steady-state the wage share is now lower), and the national income accounting restriction is given by $1 = \pi + \psi = \pi_N + \phi + \psi$.

[11] In their models however the implications on distribution is different. It is the capitalist household that sees a decline in wealth due to the transition to a funded system. Here, since the tax on profits is transferred entirely to retirees for consumption purposes, it is the wage earners who in fact end up substituting with their wage for the retirees' income.

capitalists motivates the latter to search for labor-saving technology. With $\gamma'[\psi] > 0$ and $\chi'[\psi] < 0$ the shift of production towards labor-augmenting technology places the economy in a transitional period of rising labor productivity and falling capital productivity. If the period during which labor productivity grows at a more rapid pace coincides with the demographic transition (or the period of rapid aging) the process described above may help the economy deal successfully with its aging population. However, at the new steady-state both the employment rate and the level of labor productivity are lower whereas income distribution, profit rate and capital productivity remain unchanged. The mechanism of endogenous technical change is at the heart of these results. First, income distribution is now determined exclusively by the specification of the technical progress function. Second, the concavity of the technical progress function eventually forces a slower labor-saving technical change as the system goes through the dynamics of the (Goodwin) cycle.

The presence of endogenous technological change adds new powers to a redistributive fiscal policy. If a tax is applied directly on profits, capitalists are unable to hand down the entire tax burden to wages. As Julius (2005) shows, profits and wages share the tax in a proportion given by the slope of the technical progress function. Building on section (3.3) the wage share in a Harrod-neutral economy with endogenous technical change must fulfill:

$$\psi^* = \frac{1}{1 - f'[0]} \tag{33}$$

A tax on profits, τ, renders the share of net profits as a function of the slope of the technical progress relation:

$$\pi_N^* = (1 - \psi^*)(1 - \tau) = \frac{-f'[0](1 - \tau)}{1 - f'[0]} \tag{34}$$

The tax on profits does not affect the rate of profit due to the endogeneity of ϵ_K, which adjusts to compensate for the decline in the disposable profits.

We examine now returns to retirees based on the two fiscal schemes, taxing wages and taxing profits. Based on the former, the pension level over time is obtained from (28), with the only difference that the growth rate of labor productivity is a positive function of the wage share, $\gamma'[\psi] > 0$.

$$p_\rho(t) = \frac{\rho w^* L^*}{R} = \frac{N_0 w_0}{R_0 \delta_1} e^{(\gamma[\psi^*] + n - n_r)t} (\rho\gamma[\psi^*] - \delta_2 \rho^2) \tag{35}$$

If fiscal policy derives retirees' income from profits (we therefore set $\rho = 0$), the pension level is $p_\tau = \tau\Pi/R$. Writing output as $X = \epsilon_L L$ and labor productivity as $\epsilon_L = \epsilon_{L0} e^{\gamma[\psi^*]t}$ we can solve for the pension sequence:

$$p_\tau(t) = \frac{\tau\pi X}{R} = \frac{N_0 \epsilon_{L0}}{R_0 \delta_1} e^{(\gamma[\psi^*] + n - n_r)t} \gamma[\psi^*] \frac{-\tau f'[0]}{1 - f'[0]} \tag{36}$$

Compare now (35) with (36). Simplifying the common terms we are left with $p_\rho(t) = w_0 \rho(\gamma[\psi^*] - \delta_2 \rho)$ and $p_\tau(t) = \epsilon_{L0} \gamma[\psi^*](-\tau f'[0])/(1 - f'[0])$.

First thing to notice is that a higher τ always benefits retirees, although, as we will see in a moment, at the expense of workers. Not the same is true for an upward change in the social security tax. We can verify that p_ρ responds positively to ρ as long as $\rho < \gamma[\psi]/2\delta_2$. Even though fiscal policy is now more successful one should be careful in translating this finding into an advocacy for heavily taxing profits. A quick perusal of the model's equations indicates that a higher τ sets off the economy on a transitional path characterized by a lower employment rate, a lower $\gamma[\psi]$ and a lower labor share.[12] This adjusting period can be extremely unsettling for workers who, for a given period, end up with fewer jobs and lower wages. In contrast to the model in section (4.1) a tax on profits does not change the steady-state configuration of before-tax income shares which are predetermined by the mechanism of induced technical change. After-tax profit share is different and follows from (34). After-tax and before-tax wage shares are equal and come from (33).

Secondly, retirees benefit from a shift in the distribution of income towards wages when their income is based on a social security tax. When ψ increases capitalists respond with labor-augmentation techniques which raises p_ρ. Alternatively, when the wage share rises and pensions come from profits p_τ declines.

Thirdly, induced technical change translates favorably for retirees when labor-augmenting techniques are chosen in both fiscal schemes. Higher labor productivity always helps the economy manage in a less disruptive manner its liabilities towards the retired population.

5 Endogenous social security tax

When policy sets the social security tax an increase in the relative number of retirees automatically triggers a decline in p and therefore a lower consumption level, cr^r. A different case can be made for either fixing the pension at a level $\bar{p}$ or allowing it to grow at the same rate as the real wage.

The tax rate ρ follows from (6) and can be written as $\rho = (\bar{p}R/N)(N/X)(X/wL)$. This artifice allows us to formulate the tax rate as $\rho = d_e/(l\psi)$, where $d_e = \bar{p}R/\epsilon_L N$ is the *potential* economic dependency rate that is attained if the labor force is fully employed or $L = N$. Differentiating ρ and assuming a constant level for the individual pension we obtain:

$$\hat{\rho} = \hat{d}_e - \hat{\psi} - \hat{l} = n_r - \gamma - n - \hat{\psi} - \hat{l} \tag{37}$$

At the steady-state it is always true that $n_r = n$. If the distribution of income and the employment rate are stationary the tax rate remains unchanged at the steady-state only when labor productivity is zero. When $\gamma > 0$ it is reasonable to assume that the pension level grows at the same rate as the labor productivity and wages or $\hat{p} = \hat{w} = \gamma$. [13]

5.1 Exogenous technical change and an adjusting tax - Model C

When technical change is exogenous the dynamics of our economy depend on $\hat{d}_e = n_r + \hat{p} - \gamma - n$ together with the laws of motion for the employment rate, e and the wage share, ψ.

$$\frac{\dot{\psi}}{\psi} = \delta_1 l + \delta_2 d_e/l\psi - \gamma \tag{38}$$

$$\frac{\dot{l}}{l} = (1-\psi)\epsilon_K + \chi - \gamma - n \tag{39}$$

$$\frac{\dot{d}_e}{d_e} = n_r + \hat{p} - \gamma - n \tag{40}$$

[12] The magnitude of changes that these labor variables suffer depends on the slope of technical progress function.

[13] (37) becomes $\hat{\rho} = n_r + \hat{p} - \gamma - n - \hat{\psi} - \hat{l}$.

An endogenous tax rate makes the existence of an economically meaningful steady state conditional on $\gamma - \hat{p} = 0$.[14] Assuming that this rule is satisfied and technical change is Harrod-neutral the above system reduces to equations (38)and (39). The steady-state wage share, $\psi* = 1 - (\gamma + n)/\epsilon_K$, declines with faster labor productivity and labor supply growth, and expands when capital productivity increases. The fixed point for l^* is solved from the nullcline for the distributive curve $\dot{\psi} = 0$:

$$\delta_1 l + \delta_2 \frac{d_e}{\psi^* l} - \gamma = 0 \tag{41}$$

(41) solves for two different solutions for l^*:

$$l^*_{1,2} = \frac{\gamma \pm \sqrt{\gamma^2 - 4\delta_1\delta_2 d_e/\psi^*}}{2\delta_1} \tag{42}$$

To make economic sense we need $0 \leq l^* \leq 1$. Stability analysis presented in appendix B suggests the existence of a Hopf bifurcation at the point $\gamma = -2\sqrt{\mu}$. Figure 2 signals this bifurcation by the vertical dotted line, to the left of which the system has no steady-state. When $\gamma > 2\sqrt{\mu}$ there are two positive fixed points with the larger equilibria (in algebraic terms) stable and the smaller one unstable.

[**Figure 2 here**: Endogenous social security tax and exogenous productivity]

The steady-state income distribution remains unaffected by the social security tax. It is the employment rate which adjusts when d_e rises. In figure 2 a higher d_e shifts the distributive curve to the right which shrinks the distance between the two equilibrium points. In a sense a higher dependency rate affects the steady-state l in the same direction as a higher tax rate set by fiscal policy did in the previous models. There is however an important difference between the behavior of the economy in this model compared to model A. For plausible values for the initial conditions preliminary simulation results suggest that an endogenous social security tax ensures that the economy reaches the steady-state and notably it converges to the higher equilibrium point for l.

The model behaves in a similar fashion as model A when γ changes. A higher rate of labor productivity shifts both $\dot{l} = 0$ and $\dot{\psi} = 0$ to the left. The steady-state wage share declines but the employment rate benefits as the stable equilibrium point of l^* shifts.

The overall results indicate that if the economy is not at extremely low levels of employment (see simulation results in next section) social conflict does not preclude the economy to reach employment bliss and maintain living standards for retirees. Not the same is true for workers who at the macro level are the losers in this story. As labor productivity grows they have to accept a shift of income towards profits if they want to keep their jobs. However at the individual level wages grow at a faster rate, and at the new steady-state wages expand at the same pace as labor productivity.

5.2 Induced technical change and an adjusting tax - Model D

This last model accounts for both an endogenous social security tax and induced technical change.

[14]When $\gamma - \hat{p} > 0$ the tax rate eventually goes to zero and the system becomes the regular Goodwin model of cyclical growth. If $\gamma - \hat{p} < 0$ the system does not have an economically meaningful fixed point.

$$\frac{\dot{\psi}}{\psi} = \delta_1 l + \delta_2 d_e/\psi l - \gamma[\psi] \tag{43}$$

$$\frac{\dot{l}}{l} = (1-\psi)\epsilon_K + \chi[1-\psi] - \gamma[\psi] - n \tag{44}$$

$$\frac{\dot{\epsilon_K}}{\epsilon_K} = \chi[1-\psi] \tag{45}$$

$$\frac{\dot{d_e}}{d_e} = n_r + \hat{p} - \gamma[\psi] - n \tag{46}$$

The model has a steady-state only if $\hat{p} = \gamma[\psi]$.[15] The wage share evolves to guarantee that both the economic dependency rate and capital productivity are stationary. The system must evolve towards Harrod neutrality; an outcome warranted by the induced technical change mechanism. At the same time, the level of pensions must increase at a rate consistent with labor augmentation. There is no mechanism that allows us to solve for a unique equilibrium solution without this binding constraint.

Conceptually, the above condition is similar to what we encountered in model C. Upcoming simulation results highlight however an important difference between this model and model C. As a preview, notice that the mechanism of induced technical change makes the dependency rate a function of the wage share. A higher dependency rate due to a transitional larger n_r raises ψ which further boosts labor productivity growth and therefore stabilizes the dependency rate. It follows that even if population aging is a process that will be with us for many decades to come and therefore the dependency rate does not reach a steady-state any time soon, its magnitude and rate of change will be kept under control to ensure the sustainability of the social security system over a period that is sufficiently enough. How long? Demographic forecasts mentioned above predict that population aging in most of the industrialized world will be slowing down considerably after 2050. Thus, the mechanisms in place have to help the economy deal with the effects of higher dependency rates for the next four decades. Can the mechanisms discussed here deliver this scenario? The next section goes over numerical simulations that suggest ways for the economy to achieve this state of "pseudo-bliss". Meanwhile, the analytical results presented in this section highlight the fact that the qualitative features of this pseudo-steady state are important for policy and policy space. A parameter configuration which delivers a slow growing dependency rate is a powerful tool for near future efforts to avert the so-called *financial train wreck* of the social security pension system.

6 Simulation results and concluding remarks

This paper introduces population aging in Goodwin's model of cyclical growth. It explores how changes in the secondary distribution of income due to rising old-age dependency rates may affect relations of production in capitalist economies. The paper differentiates among four models based on two types of fiscal policy — a fixed social security tax and adjusting benefits, and an endogenous tax rate and a fixed benefit level— and two technical progress regimes – exogenous and induced technical progress.

I have previously warned the reader that the Goodwin model can be seen as a tool for intuition rather than as a description of a particular economy. The models I have put together

[15] Assuming that $n_r = n$ at the steady-state.

in this paper are only a first and modest step towards addressing contemporary political economy issues related to population aging using a non-neoclassical approach. They can surely be extended to fit a particular economy or fiscal regime. They can also be augmented with more rigorous microeconomic foundations that would allow one to ask further important questions. But for now several analytical results I think are noteworthy.

Class conflict and relations of production. An increase in the social security tax in response to rising old-age dependency rates leaves primary distribution of income unchanged but reduces the rate of employment. From retirees' point of view an optimal tax rate ρ^* reminiscent of a neo-Laffer curve maximizes the level of the individual pension. Retirees would be unwise to escalate the conflict beyond this ρ^* since the ensuing rise in unemployment will work against them. Unemployment therefore stabilizes not only the worker–capitalist conflict but also the worker–retiree conflict.[16]

Fiscal Policy. After–tax profit share is unaffected by the choice of fiscal policy – taxing wages or profits – when technical progress is independent. Fiscal policy is therefore ineffective with respect to burden sharing, but has different implications for the average rate of employment at equilibrium. Specifically, a tax on wages brings down the steady-state rate of employment through the retiree-worker-capitalist chain of conflicts. A tax on profits on the other hand only changes before-tax distribution of income between wages and profits but does not alter the rate of employment.

Technical Progress. Technical progress can temporarily improve economic sustainability of population aging. Class conflict and induced technological change combine to ease the requirements on the economy: a higher tax rate leads workers to bargain for higher wages which then induces capitalists to search for labor-saving techniques. Redistributive fiscal policy is more effective when technical change is endogenous. Income distribution is determined by the Harrod-neutrality condition applied to the mechanism of induced technical change and capitalists and workers split up society's responsibility towards retirees. Specifically, transfers to retirees are shared by wages and profits in proportion to their shares in national income. Employment rate is unchanged when profits are being taxed which is not the case when ρ increases.

I conclude with a few numerical simulations which I hope complement the above formal analysis.[17] Model A captures pure predator-prey dynamics between income distribution and accumulation. An exogenous social security tax and a cyclical behavior of this economy puts retirees' income at the whims of economic fluctuations. Figure 3 tells this story.

[**Figure 3 here**: Dynamics in an economy where technological change is exogenous and fiscal policy sets the social security tax]

Simulations using model B confirm that a mechanism of induced technological change drives the economy to a steady-state as observed in figure 4. For retirees this dynamic means less volatility in their income flow. The growing divide that appears between the level of wage and the level of pension is bound to happen as long as the number of retirees expands at a rate faster than that of the labor supply, the case in this exercise.[18]

[16]The model can also be understood and modified by having retirees and workers combined into one class with heterogeneous time horizons. Certainly, higher pension benefits ratified at the present time are a gain for current workers in the future when they become retirees.

[17]Appendix C discloses values for the parameters and initial conditions upon which the four versions of the model are being solved.

[18]In reality demographic changes are expected to reach a steady-state. For the sake of exposition in this

[**Figure 4 here**: Dynamics in an economy where technological change is endogenous and fiscal policy sets the social security tax]

The dynamics of the economy with an endogenous social security tax and exogenous productivity are captured by figure 5. Starting from plausible initial conditions the economy converges in a cyclical fashion to the upper, stable equilibrium point. Also notice that the tax rate ρ converges to a constant value. It is not the dynamics of the model alone which ensures this outcome. I remind the reader that at the steady-state demographic changes stabilize $n = n_r$ and that pensions increase at the same rate as the level of wage and labor productivity. It is the transition period to the new demographic steady-state which poses economic difficulties. Policy can target productivity growth such that old-age dependency rates are kept under control.

[**Figure 5 here**: Dynamics in an economy where technological change is exogenous and the social security tax is endogenous]

Initial conditions matter a great deal for the simulations using the last model and which are viewed in figure 6. They are based on an initial value for capital productivity of $\epsilon_K = 0.2$. For the set of parameters chosen Harrod neutrality solves to $\psi = 0.67$ and the level of capital productivity from the $\dot{l}$ equation is 0.15. Pseudo-stability is more easily achieved when the economy is above this 'steady-state'level for ϵ_K which means that the mechanism of induced technical change moves the economy towards lower capital productivity and therefore higher labor productivity growth which is required to stabilize the old-age dependency rate.

[**Figure 6 here**: Dynamics in an economy where both technological change and the social security tax are endogenous]

Appendices

A Stability analysis and Jacobians

Stability analysis and Jacobian of model B. Evaluated at the fixed points the Jacobian for the system in section 4.2 is:

$$\begin{pmatrix} -\gamma^{'}\psi^{*} & \delta_1\psi^{*} & 0 \\ (-\epsilon_K^{*} - \chi^{'} - \gamma^{'})l^{*} & 0 & (1-\psi^{*})l^{*} \\ -\chi^{'}\epsilon_K^{*} & 0 & 0 \end{pmatrix}$$

The Routh-Hurwitz conditions require that the trace and the determinant are both negative. This can be easily proved. In addition we need to show that the sum of the determinants of the principal minors of the Jacobian, $Det[J]$ is larger than zero. This boils down to $-(-\chi^{'}\epsilon_K^{*}(1-\psi^{*})l^{*} > 0$, a condition which indeed is met. Finally, we need to show that $-Tr[J](\sum Det[J_i]) + Det[J] > 0$. This last criterion is met as long as the marginal effect of the wage share on labor productivity growth, γ', is larger than the effect of the employment rate on the growth rate of the wage share as captured by δ_1.

exercise no dynamics is applied to the growth rate of population n or of retirees n_r, such that they eventually converge.

*Stability analysis and Jacobian of model C.*The Jacobian of model C evaluated at the fixed points is:

$$\begin{pmatrix} -\frac{\delta_2 d_e}{l^* \psi^*} & \delta_1 \psi^* - \frac{\delta_2 d_e}{l^* \psi^*} \\ -\psi^* \epsilon_K & 0 \end{pmatrix}$$

The trace is negative and equal to $-\frac{\delta_2 d_e}{l^* \psi^*}$, while the determinant is $(\delta_1 \psi^* - \frac{\delta_2 d_e}{l^* \psi^*})\psi^* \epsilon_K$. Setting $\mu = \delta_1 \delta_2 d_e / \psi^*$, the system can have an equilibrium only where $|\gamma| > 2\sqrt{\mu}$ which suggests the existence of a Hopf bifurcation at the point $\gamma = 2\sqrt{\mu}$[19]. If $\gamma = 2\sqrt{\mu}$ the system has only one equilibrium point at $[l^*_{1,2} = \gamma/2\delta_1, \psi^* = 1 - (\gamma + n)/\epsilon_K]$ and its qualitative features resemble the permanent oscillations observed in the Goodwin model and illustrated in figure 1. When $\gamma > 2\sqrt{\mu}$ there are two positive fixed points with the larger equilibria (in algebraic terms) stable and the smaller one unstable as shown below.[20]

In terms of the the dynamic features of this system, the nullcline for the distributive curve is $\psi = \delta_2 d_e / l(\gamma - \delta_1 l)$, and has a minimum at the point where the denominator is the largest. Simple calculus finds that along the nullcline, ψ is the smallest at the point where $l = \gamma/2\delta_1$. But notice that using this value for l into the nullcline for the distributive curve solves to $\gamma^2 = \delta_2 \delta_1 d_e/\psi$, which is the bifurcation point mentioned above. To the right of the bifurcation point, the distribution of income moves in favor of wages if $\psi < \delta_2 d_e / l(\gamma - \delta_1 l)$, which visually takes place in the area outside the distributive curve. The wage share declines when the economy is situated inside the nullcline.

It has been established above that the trace is always negative. The determinant on the other hand can take either sign depending on the slope of the distributive curve which is determined by the sign of the term a_{12} in the Jacobian. The determinant is positive when the distributive curve has a positive slope or $a_{12} > 0$. In this case the system is stable as indicated by the upper equilibrium point in 2. When $a_{12} < 0$ the determinant is negative and therefore the equilibrium is a saddle point.

Jacobian of model D. The Jacobian for model D is:

$$\begin{pmatrix} -\gamma' \psi^* - \delta_2 d_e / l^* \psi^* & \delta_1 \psi^* - \delta_2 d_e / l^{*2} & 0 & \delta_2 / l^* \\ (-\epsilon^*_K - \chi' - \gamma') l^* & 0 & (1 - \psi^*) l^* & 0 \\ -\chi' \epsilon^*_K & 0 & 0 & 0 \\ -\gamma' d^*_e & 0 & 0 & 0 \end{pmatrix}$$

The system has one steady-state when $\gamma[\psi^*] - \hat{p} = n_r - n$ and Harrod-neutrality holds. Capital productivity follows from $\dot{l} = 0$ and is $\epsilon^*_K = (n + \gamma[\psi^*])/(1 - \psi^*)$. The dependency rate solves based on the initial conditions. Finally, l^* comes from (43).

B Equations and parameters used in the simulations

To avoid cluttering the text of the paper I have not included the discount factor of capitalists, β and a constant that usually appears in the wage curve. Simulations results include these two parameters and the equations for the laws of motions of wage share and employment rate used are $\dot{\psi}/\psi = \delta_1(l - 1) + \delta_2 \rho - \gamma$ and $\dot{l}/l = (1 - \psi)\epsilon_K + \chi - \beta - \gamma - n$.

I have used the following two linear equations for $\hat{\epsilon}_L$ and $\hat{\epsilon}_K$ when technological change is endogenous (models B and D):

[19] Another bifurcation point would exist at $\gamma = -2\sqrt{\mu}$ but this case is ignored as it does not meet the requirement that $l^* > 0$.

[20] For a mathematical proof of this claim in such models see Medio and Lines (2001).

$$\gamma[\psi] = -0.035 + 0.1\psi \tag{47}$$

$$\chi[\psi] = 0.04 - 0.06\psi \tag{48}$$

When technological change is exogenous (models A and C) labor productivity grows at 2 percent per period and capital productivity is 0.5, $\gamma = 0.02; \epsilon_K = 0.5$.

The other parameters have the following values for models A through C: $\delta_1 = 0.05; \delta_2 = 0.2; n = 0.02; n_r = 0.04; \beta = 0.1$.

When social security tax is exogenous I set it to $\rho = 0.05$. Dependency rate, d_e in model C is assumed to be 0.05.

For model D, all the parameters have stayed the same besides the growth rate of retirees which was raised to $n_r = 0.05$.

C Figures

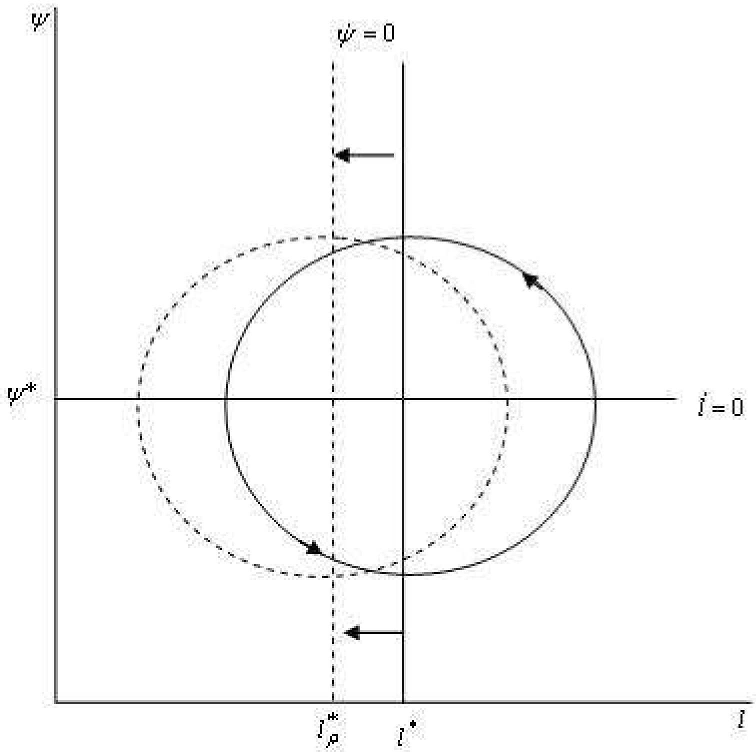

Figure 1: Exogenous social security tax and exogenous productivity

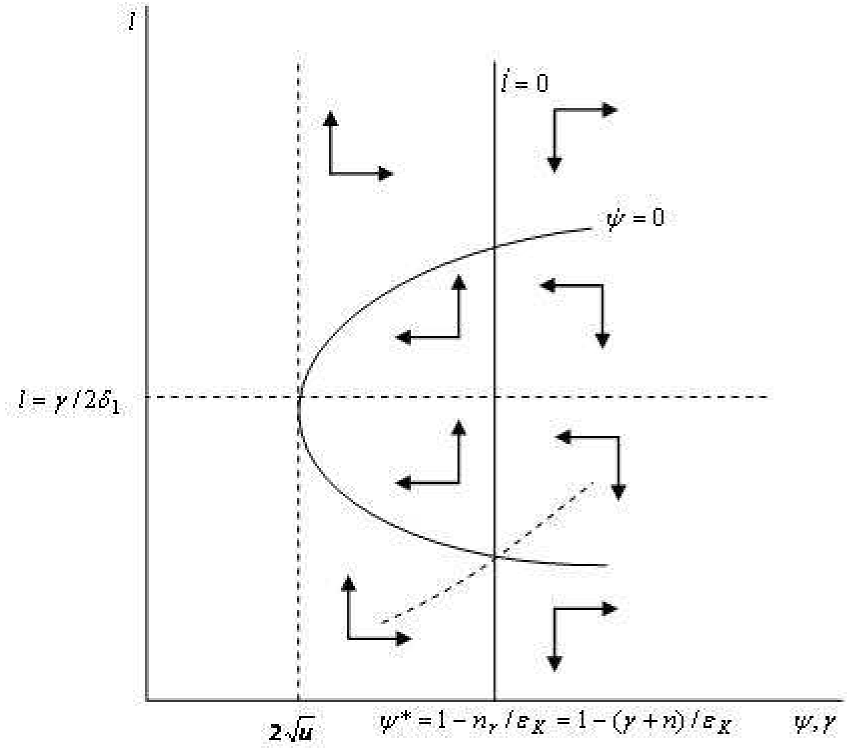

Figure 2: Endogenous social security tax and exogenous productivity

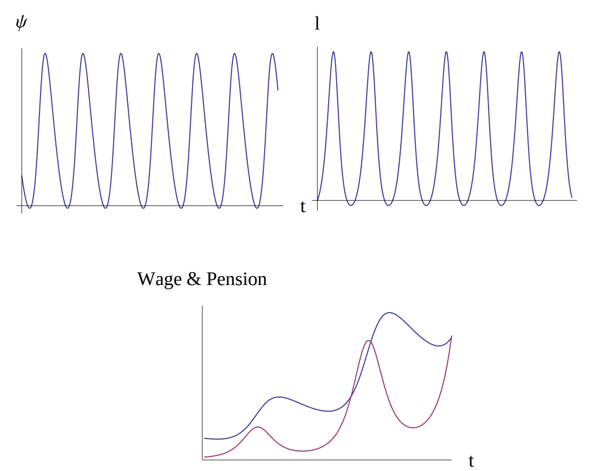

Figure 3: Dynamics in an economy where technological change is exogenous and fiscal policy sets the social security tax

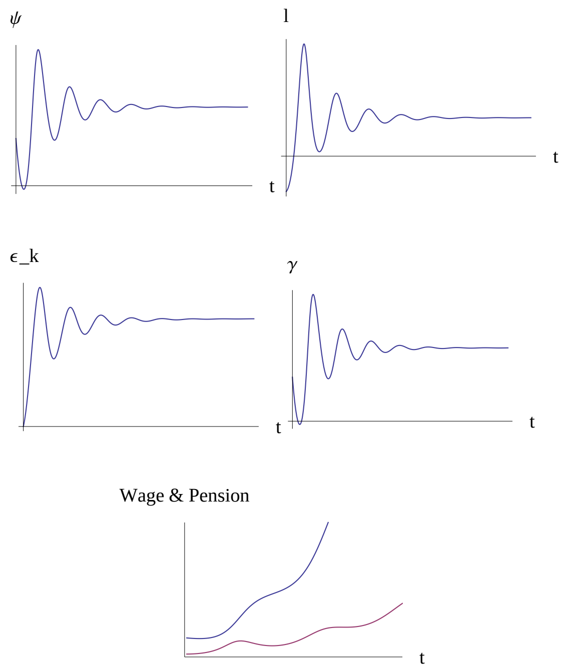

Figure 4: Dynamics in an economy where technological change is endogenous and fiscal policy sets the social security tax

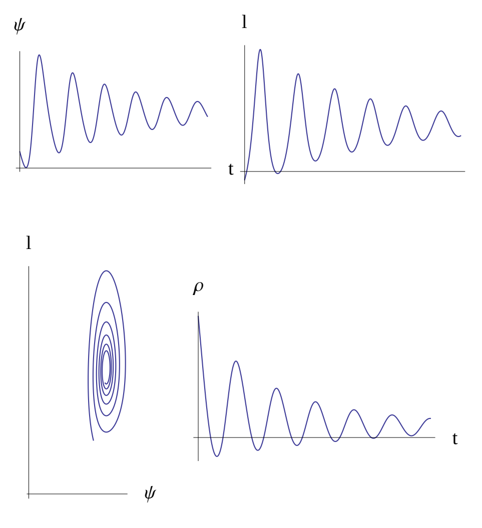

Figure 5: Dynamics in an economy where technological change is exogenous and the social security tax is endogenous

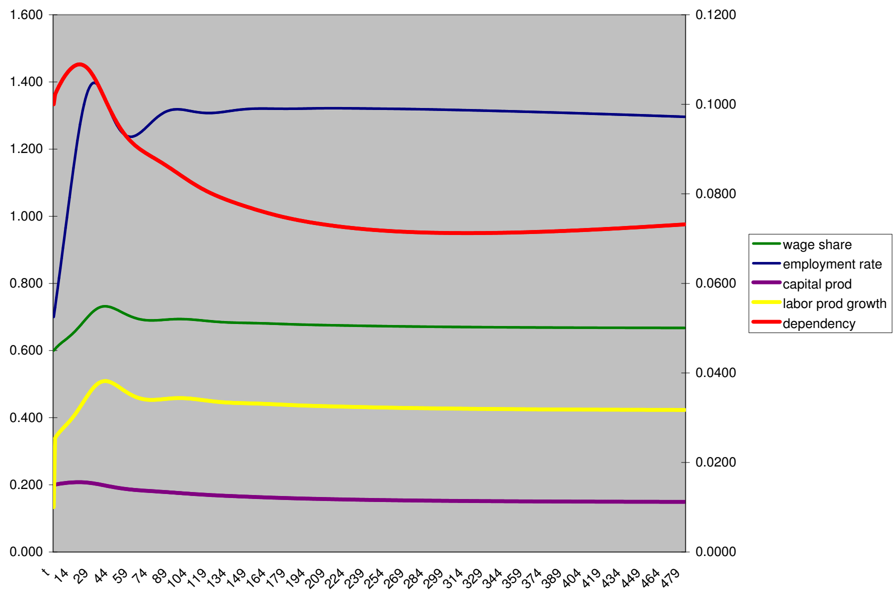

Figure 6: Dynamics in an economy where both technological change and the social security tax are endogenous

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