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# Total Factor Productivity and Income Distribution: A Critical Review

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**Total Factor Productivity and Income Distribution: A Critical Review**

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## Abstract

The aim of the present paper is to critically reappraise the validity and the relevance of the notion of total factor productivity (TFP) as a measure of technological progress. Placing the focus on the role that the neoclassical distribution theory plays in measuring technological progress, we take up the recent revival of the tautology argument (Felipe & McCombie 2003) and the simple results of the capital controversies. First, I argue that the measure of TFP exclusively relies on the marginal productivity theory of distribution through which factors' income shares are linked to their technological progress. Second, it will be shown that the marginal productivity theory of distribution is based on extremely limited theoretical and empirical grounds. Third, therefore, it is concluded that the measure of TFP as a measurement of the contribution made by technical progress to the economic growth has very little to do with the reality.

**Keywords**: Total Factor Productivity, Marginal Productivity Theory of Distribution, Income Accounting Identity, Capital Controversies
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## 1. INTRODUCTION

The growth experiences in the East Asian countries for a few decades after the second World War has drawn considerable attention of economists as well as policy makers all around the world, giving rise to a new coinage of the "East Asian Miracle" (World Bank, 1993). Indeed, the revival of growth theory in the late 1980s comes with the efforts to explain those phenomenal economic achievements that the East Asian countries had enjoyed for almost three decades. Growth economists have well documented extensive empirical research on these countries for more than a decade[1]. Miraculous they may be, economists have witnessed a super-miracle that has taken place in China for the last three decades. The Chinese economy has outperformed the East Asian Tigers during its reform period, with the average annual growth rate GDP being 9.5% between 1979 and 2004. As with the case of the East Asian Tigers, a number of empirical studies have been devoted to identifying and quantifying the growth sources in China.

It is of interest in a methodological perspective to notice that, even though the results are not conclusive and debates among economists on the major factor that might have led to the outstanding economic performance are going on, they shares a common methodology. With a few exceptions, for instance, almost all researches on sources of economic growth in China have been organized in the so-called growth accounting framework[2]. As in the investigations of the East Asian miracle, although each study uses slightly different variables and methods of their measures, the empirical studies exclusively rely on the notion that a distinction could be made between growth resources due to technical progress and those due to an increase in the inputs of the factors of production. As a result, the single most important focus of discussion about the validity of studies

[1] For the accumulation view, see Collins and Bosworth (1997), Kim and Lau (1994, 1995), Krugman (1994), Young (1992, 1994, 1995). For the assimilationist view see Klenow and Rodriguez-Clare (1997), Hsieh (1999, 2002), Sarel (1997).

[2] See, among many others, Jefferson et al. (1999, 2000), Hu and Khan (1997), Chow (1993), Chow and Li (2002), Young (2003), Ezaki and Sun (1999), Woo (1996), Wu (2003)

has been placed on the issue of the reliability of data used in the studies. In this framework, special attention is paid to the part of economic growth attributable to technical progress because it is believed that it measures the economy's efficiency.

To quantify it, Solow (1957) developed the notion of total factor productivity (TFP) which is still in extensive use in the studies on the sources of economic growth in economies including East Asian countries and China as well as other developed countries. More than three decades ago, however, it was demonstrated that, but unfortunately has ignored in the neoclassical literature, the neoclassical measure of TFP is a tautology (Phelps-Brown, 1957; Simon & Levy, 1963; Simon, 1979; Samuelson, 1979; Shaikh, 1974, 1980). As shown in short, in the growth accounting framework the distributional configuration in an economy automatically guarantees the supposed technological picture of that economy, since the measure of TFP originates from an income accounting identity of the economy. This tautological property of the measure of TFP implies not only that the alleged TFP has nothing to do with the putative technological reality until one could secure the mechanism through which the underlying technological reality manifests itself in the income distribution, but also that it is not testable in this framework because of the very definition of the tautology itself.

The aim of the present paper is to critically reappraise the validity and the relevance of TFP as a measure of technological progress. Placing the focus on the role that the neoclassical distribution theory plays in measuring technological progress, we take up the recent revival of the tautology argument (among many, Felipe & McCombie, 2003) and the simple results of the capital controversies. First, I argue that the measure of TFP exclusively relies on the marginal productivity theory of distribution through which factors' income shares are linked to their technological progress. Second, it will be shown that the marginal productivity theory of distribution is based on extremely limited, if any, theoretical and empirical grounds. Third, therefore, it is concluded that the measure of TFP as a measurement of the contribution made by technical progress to the economic growth has very little to do with the reality.

The present paper is organized as follows. Following this introduction, section 2 critically re-examines so-called growth accounting and shows that the measure of TFP is based exclusively upon the marginal productivity theory of distribution, the neoclassical theory of distribution. It is argued that the concept of TFP is a tautology unless the marginalist distribution principle is well justified. Section 3 is concerned with the implications of the capital controversies for the neoclassical distribution theory. It is shown in this section that the marginalist distribution principle would hold only under the extreme conditions which is not likely to be satisfied except for a fluke in the real world. Section 4 is devoted to proving that the econometric estimations do not provide any support for the neoclassical distribution theory because of the underlying identity. Section 5 concludes that the measure of TFP would not be a valid measurement of contribution made by technological progress.

## 2. THE NATURE OF GROWTH ACCOUNTING

To explain the long-term trend of economic growth, the growth accounting approach divides growth sources into two components: one which could be explained by the growth of the amount of input used up, and the other (remainder) that might be explained by the improvement of the efficiency in using factor inputs. The concept of TFP has been developed to measure the second component. Taking a functional relationship between net output (value-added), factor inputs, and technical progress as $Y = F(X, A)$, where $Y$ is output, $X$ is the total amount of factor inputs and $A$ is technical progress, TFP is supposed to measure the magnitude of the shift of a production function supposedly caused by technical progress, which is contrasted with a movement along the production function that might have been caused by an increase in inputs.

After having laid out the theoretical framework of the neoclassical economic growth theory in 1956, Solow (1957) demonstrated its applicability. Since then, the suggested approach to measuring TFP has taken a prerogative of a standard. This first version of the measure of TFP is called the "primal approach". A practical formula for TFP can be derived from an aggregate production function.

(1) $$Y(t) = F(X(t)_i; t)$$

where $Y(t)$ represents net output (value added) at time $t$, $X(t)_i$ indicates inputs of production factor $i$ at time $t$ and the argument $t$ is included in the functional form separately to allow for technical progress as a shift factor[3]. Assuming the argument $t$ is independent of input factors $X(t)_i$, in other words, assuming Hicks-neutral technical progress[4], the shift factor for technical progress can be factored out of the function.

(2) $$Y(t) = A(t) \cdot F(X(t)_i)$$

From equation (2), the technical progress is defined by the ratio of output to the contribution of factor inputs.

(3) $$A(t) = \frac{Y(t)}{F(X(t)_i)}$$

[3] The treatment of technological progress as a shift factor implies that this approach assume the exogenous technological progress. For the criticism of the neoclassical notion of exogenous technological progress, see Scott(1989) and Kaldor(1957)

[4] In fact, Solow(1957) assumes a capital-augmenting technical progress known as Solow-neutral technical progess, which is not consistent with moving from equation (1) to (2). However, the inconsistency is resolved by assuming the Cobb-Douglas production function that has a unit elasticity of substitution. In general, the distinction between technical neutralities such as Hicks, Harrod and Solow-neutrality vanishes, if the elasticity of substitution of the assumed production is unity like in the Cobb-Douglas. For a proof, see Allen (1968, p.250).

Log differentiating equation (3) with respect to time, we obtain a formula for the rate of growth of technological progress which is thought of as the measure of total factor productivity, also called the Solow Residual.

(4) $$a(t) = y(t) - \sum \frac{1}{F(X(t)_i)} \frac{\partial F}{\partial X(t)_i} \frac{\partial X(t)_i}{\partial t} = y(t) - \sum \alpha(t)_i x(t)_i$$

where the lower-case letters represent the growth rate of the corresponding upper-case letters as defined above. Because $\frac{\partial F}{\partial X(t)_i}$ is the definition of the marginal products of input $X(t)_i$, the multiplicative parameter $\alpha(t)_i = \frac{\partial F / \partial X(t)_i}{F(X(t)_i)}$ is considered to be the output elasticity with respect to input $X(t)_i$. Therefore, equation (4) shows that the growth rate of technological progress is measured by the growth rate that is not explained by increases of inputs of factors of production. When $\alpha_i$ is interpreted as the output elasticity with respect to input $X_i$, the growth rate of technological progress $a(t)$ calculated with equation (4) is called a Divisia index (Jorgenson and Griliches, 1967)[5].

It is of importance to note that the property of constant returns to scale is a preempted result derived from the marginal productivity theory of distribution. With the practical difficulty in measuring the (social) aggregate marginal productivity of each input, a series of assumptions is introduced. It is assumed that producers are profit maximizers and that markets are perfectly competitive for both products and factors of production. Under these circumstances, the factor prices are determined according to their marginal products, in which the parameters for the output elasticity with respect

[5] In practice, for the annual (discrete) time series data, the Divisia index which is developed for the continuous time series is approximated to the Törnqvist index of TFP (Diewert 1976): that is, the average of the shares between two consecutive data, $\overline{\alpha}(t)_i = (\alpha(t)_i + \alpha(t-1)_i)/2$ is used, along with $\ln(X(t)_i / X(t-1)_i)$ for factor input.

to each input take up the meaning of each input's income share in a national account. In another words, factors of production are paid their contribution to the production process. Under these assumptions, the alleged output elasticities must add up to unity, that is, $\sum \alpha_i = 1$ , by the definition of factors' shares of output.

The calculation procedure described above shows that the notion of the measure of TFP is established on the base of a distribution theory. If the marginal productivity theory of distribution does not hold, the measure of TFP will lose its ground to be a measurement of technological progress.

The second version of the measure of TFP which has been recently developed is one called the "dual approach." Arguably, the main contribution of this new approach would be the fact that it does not need such assumptions that were made to derive the primal approach in equation (4) (Hsieh 1999, 2002; Barro 1999). To show this, it begins with an income accounting identity.

(5) $$Y(t) \equiv \sum W(t)_i X(t)_i$$

Where $Y(t)$ denotes total value-added, $W(t)_i$ and $X(t)_i$ are the price and the amount of factor of production $i$, respectively, used up to produce $Y(t)$ at time $t$. Equation (5) tells us that output is divided into and distributed to factors of production contributed to the production. By taking a total differentiation and dividing through both sides by $Y(t)$, we get:

(6) $$y(t) \equiv \sum \alpha(t)_i w(t)_i + \sum \alpha(t)_i x(t)_i$$

where the lower-case letters represent the growth rate of the corresponding upper-case letters with the same definition used so far and $\alpha(t)_i = W(t)_i X(t)_i / Y(t)$ defines the income share of the factor of production $i$. In the dual approach, the measure of TFP is defined again as a residual in terms of equation (6) as follows.

$$\begin{aligned} \sum \alpha(t)_i w(t)_i &\equiv y(t) - \sum \alpha(t)_i x(t)_i \\ &= a(t)_i \end{aligned} \quad (7)$$

Equation (7) shows that the dual measure of TFP is a share-weighted average growth of factor prices. The moral underlying the dual measure of TFP seems to be that factor prices may rise only when, given a quantity of factors, outputs are rising due to technical improvement (Hsieh 2002, p.502). Therefore, the weighted-average of factor prices could reflect and approximate the extent of TFP growth.

The income shares of the factors of production were interpreted as a proxy for the social marginal productivities of those input factors in equation (4). However, in equation (6) and (7) the income share parameter $\alpha(t)$ is not interpreted in terms of the underlying assumptions, but it is the definition of the factors' income shares derived in the process of algebraic manipulation. This is the reason why it is argued that the dual measure of TFP does not presume profit maximizers or competitive markets.

The crucial question here is why the share-weighted average growth of factor prices would turn out to be the measure of TFP. Contrary to the argument that the dual measure of TFP is not dependent on the neoclassical assumptions made for equation (4), however, the share-weighted average of the growth of factor prices in equation (7) becomes the measure of TFP by comparing it to equation (4): otherwise, on what grounds would it be interpreted as the measure of TFP? (Felipe & McCombie, 2006). The two measures of TFP have an identical form[6], which implies that the dual measure of TFP could be able to substitute $\alpha(t)_i = W(t)_i X(t)_i / Y(t)$ for $\alpha(t)_i = \frac{\partial F / \partial X(t)_i}{F(X(t)_i)}$ in equation (4) if factor prices would move proportionately to their marginal productivities due to technical progress. The idea in a deep layer of replacing the factors' shares for the marginal productivity implies reintroduction through a backdoor of the same assumptions as those required for equation (4) to make it become a measure of TFP.

In sum, TFP measures *de facto* the share-weighted average of factor prices in either primal or dual measurement. As shown in equation (7), the measure of TFP in growth accounting is no more than a time path of the growth rate of wage and profit weighted each period by their income shares. How is it possible for a sort of manipulated distribution value to turn into a measure of technological progress? It is by relying on the marginalist theory of distribution that directly relates the income distribution to the technological concept of the elasticity of output with respect to factors of production. The marginalist theory of distribution would be secured by referring back again to the notion of well-behaved aggregate production functions in which all desirable neoclassical properties are preempted and hence guarantee the marginalist theory of distribution. This defines the tautological nature of growth accounting in practice. Therefore, the crucial question regarding the relevance of the notion of TFP is whether or not there exist, aggregate production functions that validate the marginal productivity theory of distribution.

## 3. THE NEOCLASSICAL DISTRIBUTION THEORY IN THE CAPITAL CONTROVERSIES

According to Samuelson (1962), the marginal productivity theory of distribution would be the case,

[6] In practice, the dual measure of TFP uses prices instead of the income shares of factors of production (the left hand side of equation (7)), while the primal measure of TFP uses output, factor shares and quantity of factor inputs (the right hand side of equation (4)). But, this replacement does not change things in terms of the methodological background on which it is built.

when what he calls the elasticity of the factor-price frontier, that is, the slope of a wage-profit curve, equals the distribution shares, and this is considered the standard requirement for the neoclassical distribution theory to hold. This statement may be better understood with the help of a simple algebraic exposition.

Begin with a standard neoclassical production function in which two factors of labor and capital are used for inputs.

(8) $$Y = F(K, L)$$

where, as usual, Y is output and K is capital and L is labor. By the assumption of homogenous of degree one, equation (8) would be reduced to the form in per-capita measure

(9) $$y = F(k, 1) = f(k)$$

The marginal productivity principle of distribution maintains that

(10) $$r = \frac{df(k)}{dk} = f'(k), \quad w = \frac{df(k)}{dL} = f(k) - k \cdot f'(k), \quad f'(k) = \frac{df(k)}{dk} > 0$$

Note that a monotonic reverse relation between capital and its price (interest rate) is secured by the assumption of diminishing marginal productivity of capital, $f'' < 0$. That is,

(11) $$\frac{dk}{dr} = \frac{1}{dr/dk} = \frac{1}{f''} < 0$$

But, this assertion tells nothing about the determination of the rate of profit and its relation with the distributional rule expressed in (10): it is just assumed. The condition for the distribution rule of equation (10) and (11) to hold would be uncovered more rigorously by looking closely at the shape of the wage-profit frontier line, as Samuelson (1962) suggests.

To get the slope of the wage-profit curve, let's differentiate interest rate and wage in equation (10) with respect to capital.

(12)
$$\frac{dr}{dk} = f''(k), \quad \frac{dw}{dk} = -k \cdot f''(k), \quad f'' < 0$$

Therefore,

(13)
$$\frac{dw}{dr} = \frac{dw/dk}{dr/dk} = \frac{-k \cdot f''(k)}{f''(k)} = -k$$

Equation (13) shows the hidden condition for the marginal productivity principle of distribution: it requires that the wage-profit frontier must be a straight linear line with its slope being equal to the capital (to be precise, capital-labor ratio). And, condition (13) is guaranteed implicitly by excluding the case of disturbance of value of capital stock brought about by a change in the distribution (Bhaduri 1966, 1969).

To demonstrate this point, let's take a simple one-sector model in which output could be used either for consumption or for capital input. In an equilibrium where total output is divided and distributed between wages and profits, we have an income accounting identity.

(14)
$$Y = rK + wL$$

where $Y$ is the single output, $r$ is the rate of profit, $w$ is the real-wage rate per unit of labor, $K$ is the value of capital stock and $L$ is labor units. Dividing through by $L$, equation (14) is rewritten in terms of per capita measurement as in the reduced form of the production function (9).

(15) $$y = rk + w$$

Next, let's take the total differential of equation (15).

(16) $$dy = r \cdot dk + k \cdot dr + dw$$

Equation (16) shows that the marginal productivity of capital equals the rate of profit, that is, $\frac{dy}{dk} (= \frac{df(k)}{dk}) = r$, if and only if $k \cdot dr + dw = 0$ or $\frac{dw}{dr} = -k$, which is equivalent to the condition of equation (13)[7].

Recall that, as found through the capital controversies (Harcourt 1972)[8], the neoclassical marginal productivity theory of distribution holds only in the case of the uniform capital-labor ratio across and within industries,[9] which is extremely hard to happen in the real world. On the distributional aspect, the unrealistic presumption of the uniform capital-labor ratio results in an effect of precluding the channel through which a change in distribution can revaluate the value of capital stock[10].

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[7] It can also be called zero price Wickselll Effect.

[8] For a recent review, see Cohen and Harcourt (2003)

[9] It would be worth noting that this is the same as, in Marx's term, "uniform organic composition of capital" that Marx assumed to avoid the "transformation problem".

[10] In the neoclassical paradigm, the isolation of the determination of the value of capital from the distribution is achieved by treating capital as a given factor of production or an endowment like lands, but not a produced means of production (Garegnani, 1987; Kurz, 1985; Sraffa, 1960, ch.3).

The implications of the uniform capital intensity can be extended further in terms of the so-called re-switching of techniques and capital reversal. The assumption of a linear wage-profit frontier also rules out the possibility of re-switching of techniques and capital reversal. In a distribution theory perspective, the phenomena of re-switching of techniques and capital reversal means that the marginal productivity of capital has very little to do with the rate of profit (Pasinetti 2000). According to the findings through the capital controversies, these abnormalities would not take place only if production techniques could be expressed by a linear wage-profit frontier with which they do not intersect each other more than once (Harcourt 1972).

To be more precise, allow us to take an economy in which both capital goods and consumption goods are produced using labor and capital equipments. Assuming no depreciation of capital and wage is paid at the end of production, the two production sectors are expressed as follows.

(17)
$$w l_1 + (r+1) p k_1 = 1$$
$$w l_2 + (r+1) p k_2 = p$$

or

(17)'
$$w l_1 + (r+1) p k_1 = 1$$
$$-w l_2 + [1-(r+1) k_2] p = 0$$

where $l_i$ and $k_i$ are labor and capital employed in sector $i=1$ (consumer good), $2$ (capital good), $w$ and $r$ are the wage rate and the rate of profit, respectively. Taking the price of consumer good as the unit of measurement (=1), $p$ represents the relative price of capital measured in terms of the consumption good. As mentioned above, introducing capital goods sector explicitly means that capital is not a given endowment, but instead it is treated as a produced means of production (Sraffa 1960). As shown in short, this opens the possibility for the value of capital stock to be influenced by change in distributions.

The system (17) or (17)' can be solved for the wage rate and the price of capital.

(18) $$w = \frac{1-(r+1)k_2}{l_1-(l_1k_2-l_2k_1)(r+1)}$$

(19) $$p = \frac{l_2}{l_1-(l_1k_2-l_2k_1)(r+1)}$$

Equation (18) is a wage-profit frontier in which the wage rate is a negative function of the rate of profit, $w = f(r)$, $f\,' < 0$. To show this, we differentiate equation (18) with respect to the profit rate and obtain

(20) $$\frac{dw}{dr} = \frac{-l_2k_1}{[l_1-(l_1k_2-l_2k_1)(r+1)]^2} < 0$$

Equation (20) shows that the wage-profit frontier has a negative slope. Furthermore, to determine the convexity or concavity of the wage-profit frontier line, we take the differential of equation (20) once more with respect to the interest rate.

(21) $$\frac{d^2w}{dr^2} = \frac{2l_2k_1[l_1-(l_1k_2-l_2k_1)(r+1)](l_1k_2-l_2k_1)}{[l_1-(l_1k_2-l_2k_1)(r+1)]^4}$$

Finally, we can show that the curvature of the frontier is not unique in a linear line. Instead, it depends on the relative capital-labor ratio between two sectors. To prove this, note that since $w > 0$, $1-(r+1)k_2 = l_2 w / p > 0$ from the equation for capital goods in system (17) and hence the bracket in the numerator $l_1 - (l_1 k_2 - l_2 k_1)(r+1) > 0$ from the equation for the consumer goods sector in system (17). Therefore, equation (21) implies accordingly

$$(22) \qquad \frac{d^2 w}{dr^2} \geq 0 \quad \Leftrightarrow \quad (a_1 k_2 - a_2 k_1) \geq 0$$

Equation (22) shows the frontier line would be convex (concave) to the origin if and only if the capital-labor ratio of the capital sector is greater (less) than that of the consumption goods sector. It is only when the two capital-labor ratios are equalized, that is, $l_1 k_2 - l_2 k_1 = 0$ or $\frac{k_1}{l_1} = \frac{k_2}{l_2}$, that the frontier can be a linear straight line, that is, $\frac{d^2 w}{dr^2} = 0$ with the slope being $\frac{dw}{dr} = \frac{-l_2 k_1}{l_1^2} = -\frac{p k_1}{l_1} < 0$ $(\because\ p = l_2 / l_1)$. This is the condition that Samuelson (1962) found. Under this too restrictive condition, the wage-profit frontier is a straight linear line and the possibility of re-switching and capital reversal is ruled out.

It is of interest to note that, under this condition, a variation of distribution would not lead to a change in the value of capital stock. In equation (19), $p = l_2 / l_1$ when $l_1 k_2 - l_2 k_1 = 0$, which implies that the relative price of capital is determined exclusively by the ratio of the labor required to produce one unit of capital goods to the labor required to produce one unit of consumption goods and, therefore, the effect of the change in the profit rate in the denominator on the relative price of the capital good is ruled out.

To conclude, according to the facts found in the capital controversies, the capital-labor ratio in an economy would have a monotonic reverse relationship with the interest rate only under the condition of uniform capital-labor ratio across the economy, which might well turn out to have very limited relevance to the real world. This finding casts much doubt about the validity of the measure of TFP in the growth accounting practice which is based on the notion of the marginal productivity of factors of production as the distribution principle.

## 4. ECONOMETRICS FOR THE PUTATIVE PRODUCTION FUNCTIONS

Encountering the various criticisms of the notion of the well-behaving aggregate production functions, neoclassical economists have tried to empirically estimate putative production functions with the help of one or another econometric models. At a first glance, the estimates look successful. As cited repeatedly, the seemingly successful econometric results are referred to in order to justify the existence of production functions in which the marginal productivity theory of distribution holds (Solow, 1966; Ferguson, 1969, Ch.12). Recently this econometric approach has been even extended to test directly the underlying hypotheses of the marginal productivity theory of distribution and constant returns to scale (e.g. Kim and Lau 1994)[11]. Reflecting the accumulated empirical estimates of the production functions, they conclude that although the underlying assumptions might be unrealistic and/or wrong, the Cobb-Douglas production functions work very well[12]. Indeed, in

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[11] Sometimes, in the literature, econometric approach is often described as an alternative to the index approach discussed in section 2 (Hulten, 2001). Because, instead of using factors' income shares that are observed in a national income account, the econometric approach estimates directly the elasticities of factors of production from data sets for inputs and outputs, it is even argued that econometric approach not only do not rely on the neoclassical assumptions, but enables researchers to test the underlying assumptions such as profit maximizer and competitive markets. However, as shown below in short, econometric practices cannot be considered to be a new approach in the sense that, except for the estimated output elasticities, it is based on the equivalent notion of TFP and the identical framework to that of the index approach. Furthermore, in the context of current discussion, it is argued that the test procedure is not valid because of the underlying identity and therefore econometrics estimates of share values do not support the presumed distribution theory.

[12] Solow (1974) provides a lucid definition of working of aggregate production functions worth quoting: "When someone claims that aggregate production functions work, he means (a) that they give a good fit to input-output data *without* the intervention of data deriving from factor shares; and (b) that the function so

almost all cases for the estimated Cobb-Douglas production functions, the estimated elasticity of output with respect to each factor of production seems to be very close to the observed income share ($\beta_1 = \alpha$ , and $\beta_1 + \beta_2 = 1$) with very high value of $R^2$.

But, in this section it is proved that the econometric estimates of the output elasticities cannot be interpreted as the marginal productivity of inputs. Contrary to the proponents' belief in the existence of the well-behaved aggregate production functions, it is proved that all estimations of aggregate production functions do is to reproduce an income accounting identity of the economy under consideration and the coefficient estimates are merely approximates of the factors' income shares. Therefore, the estimated elasticities cannot be compared with the observed factor's income shares to test for whether or not the assumptions hold. If this is the case, it is needless to say that the econometrically estimated elasticities cannot be used for the weighting factors in the calculation of TFP.

To make our exposition simpler and clearer, we will take only the case of the Cobb-Douglas production function[13] in which all factors of production are reduced to two inputs, capital and labor, we have a simple relationship between income and distributions.

(23) $$Y(t) \equiv W(t)L(t) + R(t)K(t)$$

where $L(t)$ and $K(t)$ are inputs of labor and capital at time t, respectively, and $W(t)$ and $R(t)$ are wage per unit labor and profit rate of capital at time t, respectively. Equation (23) just

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fitted has partial derivatives that closely mimic observed factor prices." (a footnote is omitted, original emphasis)

[13] The same argument can be extended without any difficulty to a bit complicated (putative) production functions such as CES and Translog functions. For more detailed discussions on the CES functions, see Felipe (2000), Felipe & McCombie (2001), and for the Translog function, see Felipe (2001) and Felipe & McCombie (2003).

shows that all produced value added is distributed to the factors of production. Note that equation (23) is an income accounting identity that holds no matter how the distribution is determined; it does not tell us the distributional rule, that is, how to determine the value of $W(t)$ and $R(t)$.

Take the total differential of equation (23) with respect to time and divide through both sides by $Y(t)$ to obtain:

(24) $$y(t) \equiv \alpha(t)w(t) + (1-\alpha(t))r(t) + \alpha(t)l(t) + (1-\alpha(t))k(t)$$

where the lower-case letters are the growth rates of the corresponding upper-case variables, and $\alpha(t) = W(t)L(t)/Y(t)$ is the labor income share and $1-\alpha(t) = R(t)K(t)/Y(t)$ is the capital income share since $\frac{W(t)L(t)+R(t)K(t)}{Y(t)} = 1$. Now suppose that the factor shares are constant so that we can drop time augment $t$. Assume further that the wage and profit rate grow at a constant exponential rate, that is, $w(t) = e^{\omega t}$ and $r(t) = e^{\gamma t}$, where $\omega$ and $\gamma$ denote the constant growth rate of the wage and profit rate, respectively. Under these assumptions, the identity (24) becomes:

(25) $$\begin{aligned} y(t) &= \alpha\omega + (1-\alpha)\gamma + \alpha \cdot l(t) + (1-\alpha)\cdot k(t) \\ &= \varphi + \alpha \cdot l(t) + (1-\alpha)\cdot k(t) \end{aligned}$$

where $\varphi(t) = \alpha\omega + (1-\alpha)\gamma$ is a constant of the share-weighted average growth of factor prices. Finally, integrate (25) and then take an anti-logarithm. Then we obtain equation (26) as follows:

$$\int \frac{dY(t)}{Y(t)} dt = \int \varphi dt + \int \alpha \frac{dL(t)}{L(t)} dt + \int (1-\alpha)\frac{dK(t)}{K(t)} dt$$

$$\ln Y(t) = \varphi t + \alpha \ln L(t) + (1-\alpha)\ln K(t) + \ln c$$

and

(26) $$Y(t) = c \cdot e^{\varphi t} L(t)^{\alpha} K(t)^{1-\alpha} = c \cdot A(t) \cdot L(t)^{\alpha} K(t)^{1-\alpha}$$

where $A(t) = \exp(\varphi t)$ and c is a constant of the integral. It is important to note that equation (26) has been derived from an income accounting identity (23) under restriction of stable income shares and a constant exponential growth rate of the wage and profit rate. Though its origin is an identity, equation (26) shows a Cobb-Douglas (production) function. A national income accounting identity can be rewritten as the Cobb-Douglas function. However, the Cobb-Douglas form of equation (26) does not imply anything about the behavioral relationship between output and the factor inputs as well as income distribution, for the putative Cobb-Douglas (production) function has been derived from an income accounting identity. The assumption of the stable income shares introduced above does not weaken this point. In effect, as shown in short, it is purely an empirical issue. That is, even without that assumption, one can derive a Cobb-Douglas type (production) function from an income accounting identity which has nothing to do with the income shares, factor prices and elasticity of output, whatsoever.

To see the relationship between the income accounting identity and econometric estimates, allow us to write a typical econometric version of a Cobb-Douglas such as equation (26).

(27) $$\ln Y_t = c + \beta_0 t + \beta_1 \ln L_t + \beta_2 \ln K_t + \varepsilon_t$$

When restricted by the assumption of constant returns to scale, that is, $\beta_1 + \beta_2 = 1$, equation (12) becomes:

$$\text{(28)} \qquad \ln Y_t = c + \beta_0 t + \beta_1 \ln L_t + (1-\beta_1)\ln K_t + \varepsilon_t$$

Equation (13) can be even further simplified in terms of variables per labor.

$$\text{(29)} \qquad \ln y_t = c + \beta_0 t + (1-\beta_1)\ln k_t + \varepsilon_t$$

In equation (29), $y_t = Y_t / L_t$ and $k_t = K_t / L_t$, but they are not the growth rate as in the previous notational rule. The subscript of t means the identical to the time augment of t in the previous equations. In econometric practice, the estimates of $\beta_1$, $\beta_2$ are taken to be the output elasticity with respect to labor and capital, respectively. And, the coefficient of the time trend $\beta_0$ is supposed to capture the growth rate of technological progress. This proves that equations (27) through (29) are the logarithmic stochastic forms of equation (26), that is an income accounting identity, in which $\beta_1 = \alpha$ , $\beta_2 = 1-\alpha$ and $\beta_1 + \beta_2 = \alpha + (1-\alpha) = 1$ by the definition of the income shares. And, the time trend term in the econometric models $\beta_0 t$ corresponds to the time path of the share-weighted average growth of the factor prices in equation (25) and (26) $\varphi = \alpha\omega + (1-\alpha)\gamma$ or $A(t) = \exp(\varphi t)$ . Therefore, it is safe to conclude that an income accounting identity founded on a national account implies even econometric versions of a Cobb-Douglas production function such as (27) though (29).

Econometric models designed to estimate the output elasticities and used to test the existence of a Cobb-Douglas production function is *de facto* a statistical version of an income accounting identity. This finding has destructive implications for the relevance of the estimated production functions. The fact that the statistical models are merely an identity implies that the econometric models cannot be refuted by data: in other words, the econometric results cannot be anything but a reproduction of the implied income accounting identity that tell us nothing about a behavioral relationship between output and factor inputs. Estimates of the econometric models in equation (27) through (29) must yield perfect fit ( $R^2 = 1$ ) with the coefficients being the implied income shares, if and only if the two assumptions ((1) stable income shares and (2) constant exponential growth of the wage and profit rate) hold. There cannot be anything else.

In practice, however, it is hard to expect a perfect fit of a model. Indeed, more often than not, an econometric practice even yields a negative value for the coefficient of capital stock. It seems that these experiences give neoclassical economists an impression that the Cobb-Douglas production function exists (Solow 1974). Quite contrary to their impressions, it can be explained in terms of a pure econometric sense. Recall that the econometric models for the putative production functions have been derived under the two crucial assumptions which might not hold well in reality. First, the income shares may not be so stable, but instead varies for some reasons over time. In this case, the econometric estimates would represent an average of the income share over the sample period. Second, more importantly, the wage and profit rate are not likely to grow at a constant exponential rate. In equations (27) through (29), as usual in almost all practical researches, a simple linear time trend $\beta_0 t$ is supposed to capture the evolution of the share-weighted factor prices over time. As shown above, this would be the case if and only if wage and profit grow at a constant rate of, say, $\omega$ and $\gamma$, respectively. But, there is no reason that the factor prices grow at constant exponential rates. Indeed, in practice real economy data usually reveal the time path of the share-weighted factor prices which is quite far from a simple linear trend. If the linear time trend fails to approximate the real data, the econometric models suffer from the so-called misspecification problem in a purely econometric sense, which might well result in unreliable estimates.

In sum, an income accounting identity implies the neoclassical production functions: in so far as an economy has a consistent national income account data, the economy is neoclassical in which the measured income shares always reveal the (putative) social marginal productivities. Econometric practices cannot help to test this axiom or the existence of the neoclassical production functions, and they cannot estimate the social marginal productivities of factors of production, because of the underlying income accounting identity. Therefore, the reason for the share-weighted average of factor prices to be the measure of technological progress (TFP) is still left to be "a statement of faith." (Ferguson 1969, p.269).

## 5. CONCLUSION

Contrary to the impression of a survey article's title, "The Neoclassical Theory of Growth and Distribution", Solow (2000) ends the 30 pages long discussion with one very short paragraph on the neoclassical distribution theory and economic growth.

> Very little has been said in this survey about income distribution (in other words, about the determination of factor prices). That is *because there is no special connection between the neoclassical model of growth and the determination of factor prices [i.e. income distribution]*. The usual practice is to appeal to the same view of factor pricing that characterizes static neoclassical equilibrium theory. If *the working assumption that all markets clear* were to be lifted, an alternative theory of factor prices would certainly be needed. Much else would change besides. (Solow, 2000, p.378, italic emphasis added)

The present paper could conclude the previous discussions by contrasting the arguments above to the viewpoints implied in this succinct comment on neoclassical distribution theory. First, "the assumptions that all markets clear" are not just working assumptions taken for the simplicity sake. It is the assumptions of profit maximizers and all competitive markets for both products and factors of production that make the entire model work. What was shown above is that they are the theoretical corner stones underpinning the "usual practice" of economic growth theory, the measure of TFP which Solow himself developed a long time ago.

Second, that is so because there is a special connection between the neoclassical growth model and the income distribution. The "working assumptions" serve as a device that turns the distribution into the technological measure (marginal productivity of factor of production). Without this connection, how would it be possible that a share-weighted average of the growth rate of factor prices is thought of as the measure of TFP? The most important result of the capital controversies lies in proving that the neoclassical distribution theory is not built on solid foundation.

Third, econometric practices turn out to have failed in providing any empirical support for the special connection between income distribution and technological progress. Because of the underlying income accounting identity, econometric practices cannot help to test the assumptions that the neoclassical theory of economic growth and technological progress is built on, and they cannot estimate the social marginal productivities of factors of production.

Finally, therefore, the weakly grounded neoclassical distribution theory makes the practice of neoclassical growth theory very limitedly attractive to the sound students of economics[14].

[14] For example, when Solow concludes that "[t]he ideological overtones of this [capital] controversy were pervasive" (Solow 2000, p.351), he means that "[m]ost, perhaps all, of the objections to the use of 'capital' as a factor of production have nothing special to do with the theory of growth." He should have been kind enough to show why they do not, instead accusing them for the "ideological overtones." If those criticisms were to be just an ideological passion, he would have to defense his theory by politics, but not by scientific rigor. Because, when critics argue that it is not "a matter of simplification" and provides why as in section 3 above, he just ignored them with no reasonable justification and repeated his assertions (in addition to his attitude here to the capital controversies, for an example of his autistic stubbornness, see the exchanges between Shaikh(1974, 1980), Solow(1974, 1987)). The circumstances in which the ignoring strategy (for a

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detailed response of neoclassical economists to the capital controversies, see Pasinetti 2000) works would not be explained by anything but the political power that the neoclassical economists enjoy.

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