There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are weighted differently from Prof. Kiefer's questions: Prof. Lozada's questions are worth 10 points, 18 points, and 8 points, while Prof. Kiefer's questions are worth 18 points, 9 points, and 9 points.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 10 points; the second is worth 18 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 8 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It is helpful (but not required) for you to put the number of the problem you are working on at the top of every page of your answers.

Good luck.

Section 1.
Answer all of the following three questions.

1. [**10 points**] Suppose a profit-maximizing competitive firm uses two inputs, x_1 and x_2, to produce output according to the production function $y = f(x_1, x_2)$. Under what circumstances will increases in w_1, which is the price of the first input, cause output to fall?

2. [**18 points**] Suppose an economy consists of two price-taking persons, "a" and "b". Person a has available 1 unit of "time" which he divides between rest R_a and labor l_a. Person b has available 1 unit of "time" which he divides between rest R_b and labor l_b.

   Good "x" is produced by one competitive firm according to the production function
   $$x = \text{labor hired}\,.$$
   This firm is completely owned by Person a.

   Let the amount of good x consumed by Person a be x_a and the amount of good x consumed by Person b be x_b. Let $X = x_a + x_b$. (Although the symbol for the good, "x," and for the total amount of it produced, "X," are easily confused, only the latter plays an important role in the equations needed to work out this problem.)

   Suppose production of x causes air pollution which decreases the utility of Person b but not of Person a (perhaps because Person a lives far away from the source of the air pollution). The utility functions of the two individuals are
   $$u_a = x_a\, R_a$$
   $$u_b = x_b\, R_b - X\,.$$
   Suppose that Person b always sees "X" as exogenous; he never thinks of X as being $x_a + x_b$ (even though it is), and therefore he never considers the effect of his own consumption of x on the amount of air pollution.

   To combat pollution, the government may intervene in this economy by imposing a tax of TX on the firm, where $T \geq 0$. (So the marginal tax rate on the firm's output is T and the total tax revenue is TX.) If it does so, it gives half of the money it receives to each consumer. Neither consumer ever knows where this money comes from; each consumer thus considers it a "lump sum" gift. Call the amount of money each consumer receives from the government "t." Clearly $t \geq 0$.

Take the price of labor as the numéraire.

(a) Does the production function have increasing, decreasing, or constant returns to scale? Why?

(b) Show that, in equilibrium, $2t = TX$. (This is very easy.)

(c) Find the demand for x, supply of l, and demand for R of Person a, assuming nothing about the values of T and t except that they are not negative.

(d) Find the demand for x, supply of l, and demand for R of Person b, assuming nothing about the values of T and t except that they are not negative.

(e) Using your results so far, show that

$$T = \frac{2t}{1-t} .$$

(Hint: use the production function.)

(f) Find the general equilibrium price of x and find the amount of firm profit.

(g) Find the equilibrium demand for x of Person a, and find the equilibrium demand for x of Person b. It is fine if they are functions of t, but they should not be functions of T (you can use the result of part (e) to eliminate T).

(h) Show that in general equilibrium,

$$u_a = \frac{1-t^2}{4} \quad \text{and}$$
$$u_b = \frac{-3+4t-t^2}{4} .$$

(You can think of these as being the indirect utility functions.)

(i) Using the results of part (h), suppose the government is controlled by a social planner who wishes to maximize

$$W = \alpha\, u_a + (1-\alpha)\, u_b$$

for $\alpha \in [\,0, 1]$. Find the social planner's optimal value of t. Why does it make sense?

(j) In this problem, is the competitive equilibrium with no government intervention Pareto Optimal?

3. [**18 points**]

Required Question.

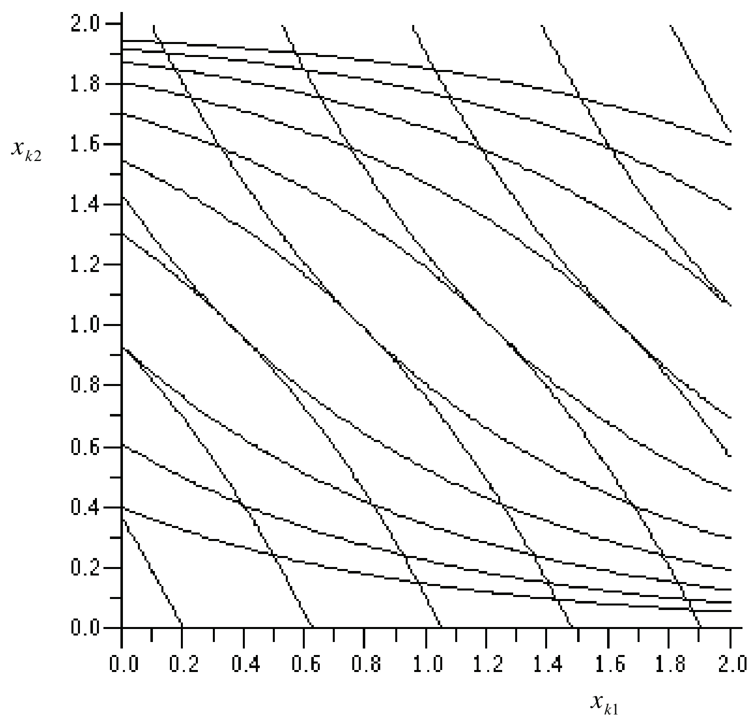
Story One: Kim and Kanye consume two private goods, coffee x_1, and croissants x_2. The utility functions and endowments are given as follows:

$$\begin{array}{lll} \text{Kim} & U_k = x_{k1} + \ln(x_{k2}) & \omega_k = (2, 1), \\ \text{Kanye} & U_w = x_{w1} + \ln(x_{w2}) & \omega_w = (0, 1). \end{array}$$

A feasible general equilibrium is described by $0 = \omega_{w1} + \omega_{k1} - x_{w1} - x_{k1}$ and $0 = \omega_{w2} + \omega_{k2} - x_{w2} - x_{k2}$. Kim and Kanye agree on the social welfare function,

$$W = \min(U_w, U_k).$$

The diagram below plots the indifference curves of both with respect to Kim's consumption bundle.



(a) Identify the endowment point and the core. Find the Walrasian equilibrium from the given endowment. What is the equilibrium price vector? Illustrate your answer.

(b) Find the Pareto set and the social optimum. Illustrate your answer. Discuss the philosophical basis for this social welfare function.

Story Two: Kim and Kanye consume a private good, coffee x_i, and a public good, poetry G. The utility functions and endowments (of coffee) are given as follows:

$$\begin{array}{lll} \text{Kim} & U_k = x_k + \ln(G) & \omega_k = 3, \\ \text{Kanye} & U_w = x_w + \ln(G) & \omega_w = 1. \end{array}$$

Both may make a contribution g_i toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i.$$

The coffee can be transformed into poetry according to the transformation function

$$0 = x_w + x_k + g_w + g_k - \omega_w - \omega_k.$$

Again, Kim and Kanye agree on the social welfare function,

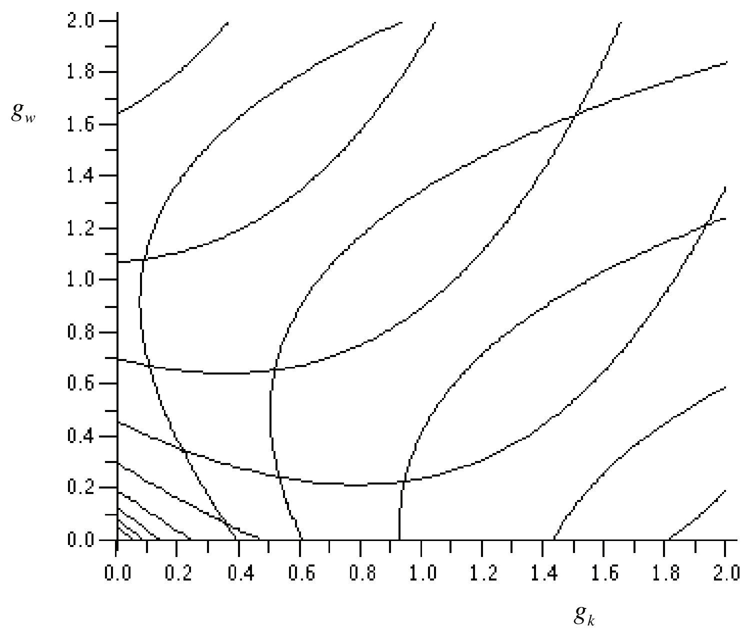
$$W = \min(U_w, U_k).$$

The diagram on the next page plots the indifference curves for both in contribution space.

(c) Show that there are multiple Nash equilibriums. Find the Lindahl equilibrium. Illustrate your answer. Consider a reform of in favor of the Lindahl equilibrium. Show that this reform is not always a Pareto improvement, but can be justified according the compensation criterion.

(d) Find the Pareto set and the social optimum.

(e) How are the two stories similar? How do they differ? Discuss how the welfare theorems (first, second and third) apply to each story.



2.0
1.8
g_w
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
g_k

Section 2.
Answer one of the following two questions.

1. [**8 points**] Consider the input requirement set

$$V(y) = \{\mathbf{x} \in \mathbf{R}^2_+ : x_1 \geq y,\ \tfrac{1}{2}x_2 \geq y\}.$$

   (a) Is this technology monotonic?
   (b) Is V convex? (Prove this formally.)
   (c) What is the production function for this technology?

2. [**8 points**] Suppose there are N price-taking (i.e., competitive) consumers, all of whom earn the same income \$$m$, all of whom consume two commodities x and y, and all of whom have the identical utility function $U = x^\alpha y^\beta$ where α and β are positive.

   Suppose there are F price-taking (i.e., competitive) producers of good x, each having the same cost function $C(x)$ for producing x.

   (a) If $N = 1$ and $F = 1$ but the agents still act competitively, and if $C(x) = x^2$, how will changes in β affect the equilibrium price of x?
   (b) If N and F are arbitrary natural numbers and if the form of $C(x)$ is unspecified (but the firms' second-order conditions are met), how will changes in β affect the equilibrium price of x?
   (c) If $N = 1$ and $F = 1$ and $C(x) = x^2$ but the agents still act competitively, how does the consumer think changes in β will affect the equilibrium price of x?

Section 3.
Answer two of the following three questions.

1. **[9 points]**
   Imagine a duopoly game with the following profit payoffs.

profit payoffs: (Google, Apple)		Apple	
		passive	aggressive
Google	passive	$(9, 6)$	$(5, 5)$
	aggressive	$(10, 1)$	$(2, -1)$

   Think of this as a nonspecific game, not necessarily Cournot or Bertrand. Only two strategies are available.

   (a) Consider a single simultaneous game. Does either player have a dominant strategy? Is there more than one Nash equilibrium?

   (b) Now consider an infinite number of repetitions of the simultaneous game. Are there any conditions under which the (passive, passive) outcome is a Nash equilibrium? If (passive, passive) is an equilibrium, is it subgame-perfect?

   (c) Suppose that Apple moves first, and that only one game is played. Draw the extensive form of this sequential game. What is the subgame-perfect Nash equilibrium? Discuss.

2. [**9 points**]
Consider the duopolistic jet plane market. Airbus and Boeing both produce jets y_i with $i = a, b$; both face marginal and average costs of 1 per jet. Suppose that the inverse demand for jets is $p = 7 - y$, where y is the total quantity of jets.

   (a) Suppose that two firms share this market, and that they behave as a Cournot duopoly. What is the equilibrium? Illustrate your answer with best-response curves.

   (b) Now suppose that these two firms behave as a Stackelberg duopoly in which Airbus chooses its quantity first, and Boeing follows. What is the Stackelberg equilibrium? Illustrate your answer.

   (c) Suppose that the EU government imposes a global-warming tax of t per plane on Airbus only. What is your intuition about the effect of this tax on the Stackelberg duopoly? Find $\partial y_a/\partial t$ and $\partial y_b/\partial t$. Illustrate your answer.

3. **[9 points]**
A democratic society consists of many citizens, identical except for their employment status. There are only two time periods: the present ($t = 1$) and the future ($t = 2$). Each individual has the following utility function,

$$U^j = E\left(2\sqrt{c_1^j} + 2\sqrt{c_2^j}\right) \qquad \text{for } j \in \{e, u\}$$

where

e denotes being employed,

u denotes being unemployed, and

c_t^j is consumption in the t^{th} period.

The unemployment rate in period 1 is $u_1 = 0.10$. The probability that an employed in period 1 will lose her job for period 2 is $\phi = 0.056$ (the firing rate), while the probability that an unemployed will gain a job is $\nu = 0.50$ (the hiring rate).

During the first period an election sets a tax τ on the employed during the second period to finance the unemployment insurance benefit f. Total tax collections equal benefits paid. In the first period employed consumption is $c_1^e = 1$, and unemployed consumption is $c_1^u = 0$; in the second period $c_2^e = 1 - \tau$ and $c_2^u = f$. On election day voters know their employment status in period 1, but not in period 2.

(a) What tax does the employed majority prefer? What is the implied benefit level?

(b) A social planner has a Benthamite welfare function. What tax and benefit level would this planner prefer?

(c) Explain why the adverse selection and moral hazard issues are not relevant in this example. Discuss the wider implications of this model for studying social conflict.