This is a take-home exam. Because of this, you can consult any books, notes, or other information available online as well as off-line. However, do not discuss the exam with anybody except for Alex or Prof. Lozada, and

ABSOLUTELY NO COLLABORATION.

Your Zoom connection to Alex can run either on your laptop/ desktop (where it will point to you, not to your desk) or on your smartphone (where it can point to the general scene of you, your computer, and your desk).

You have 4 hours and 30 minutes to finish this test. This gives you about 45 minutes per question.

While taking the test:

- *Please make sure that all your answers are legible*. Answers with illegible or difficult-to-read-handwriting may lower your grade because we may not be able to read and understand your answers. Remember that we will be reading from photos of the originals, not the originals themselves, so resolution is lost, and more resolution is lost when the answers are displayed on a screen or printed out on paper.
- *Start each question on a new sheet of paper*. For example, if your answer to Section 3's Question 1 ends near the top of a sheet of paper, leave the rest of that sheet of paper blank and start Section 3's Question 2 on a new sheet of paper. (It is OK to have multiple parts of a *single* question on a single sheet of paper, so for example parts (a), (b), and (c) could be on the same sheet of paper.)
- *Show all your work*, including work you might otherwise put on scratch paper, on your exam answer sheets. This way we can follow your entire train of thought and can inspect each one of your algebraic steps if necessary.

- *Keep your answers well organized* by numbering your answer pages at the top of each page, using either one of the following two styles:

example	*either* *this*	*or* *this*
Section 1 Question 1 page 1	1/1 p. 1	1/1 p. 1
Section 1 Question 1 page 2	1/1 p. 2	1/1 p. 2
Section 1 Question 1 page 3	1/1 p. 3	1/1 p. 3
Section 1 Question 2 page 1	1/2 p. 1	1/2 p. 4
Section 1 Question 2 page 2	1/2 p. 2	1/2 p. 5
Section 2 Question 2 page 1	2/2 p. 1	2/2 p. 6
etc.	⋮	⋮

- *Do not use different colors* in your answers because we may grade looking at black-and-white printouts of your exam.
- *To ask a question of Prof. Lozada* during the test, use the "chat" feature of Zoom to ask him a question (note that texts of such chats, even if they are only between two people, may be public after the exam ends). You can also ask Prof. Lozada to join you in a breakout room, where you can have a private voice conversation and see each other on video; he will then arrange that and have Zoom send you an invitation when it's ready.
- Prof. Dugar will not be available for questions during the test. If you think there is some ambiguity in one of his questions, be sure to spell out exactly how you are interpreting that question.

Before you send your answer pages to Alex:

- Arrange your answer pages in the order the questions appear on the exam.
- Put your Exam ID number near the page number, either on every page or at least on the first page of the answer to each question. (Remember not to put your name anywhere.)
- Also, do not discard the original answer sheets; they may be needed if questions arise about how to read something.

There are 100 points possible on this exam, 50 points each for Prof. Lozada's questions and Prof. Dugar's questions. The exam has three sections:

- In the first section contains all of the required questions. There are four of them. The first two are worth 17 points each; the second two are worth 18 points each.
- In the second section there are two questions; you should work one of them. Each is worth 14 points.
- In the third section there are two questions; you should work one of them. Each is worth 16 points.

In this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

Good luck.

Section 1.
Answer all of the following four questions.

1. **[17 points]** Denote a commodity by x, denote the supply of that commodity by x^S, and denote the demand for that commodity by x^D.

   Suppose that x is supplied by an industry whose firms take the price of x, which is denoted by p_x, given, and they also take the wage rate, which is denoted by w, given. Suppose that x^S depends on both p_x and on w. Just for simplicity, suppose that there is only one firm.

   Suppose that x is demanded by consumers who take the price of x, which is denoted by p_x, given, and they also take the wage rate, which is denoted by w, given. Suppose that x^D depends on both p_x and on w. Just for simplicity, suppose that there is only one consumer.

   This problem is inspired by study of minimum wage legislation, so we will always suppose that the market for x clears but we will make no assumption about whether the market for labor is in equilibrium or not. We wish to study the effect of an *exogenous* increase in the wage rate w, caused, for example, by an increase in the minimum wage.

   (a) Show that
   $$\frac{dp_x}{dw} = \frac{\frac{\partial x^D}{\partial w} - \frac{\partial x^S}{\partial w}}{\frac{\partial x^S}{\partial p_x} - \frac{\partial x^D}{\partial p_x}} . \tag{1}$$

   (b) Suppose the firm produces x from labor ℓ using the production function $x = 2\sqrt{\ell}$. Recalling that the price of x is denoted by p_x in this problem, and that the wage rate is denoted by w, show that the firm's supply curve for x is given by $x^S = 2p/w$.

   (c) Suppose the consumer obtains utility from consuming x, from consuming another good y, and from leisure, according to
   $$u = x \cdot y \cdot \text{leisure}\,,$$
   where leisure $= 24 - \ell$. If the consumer's only income comes from supplying labor, show that the consumer's demand curve for x is given by $x^D = 8w/p_x$.

   (d) Show that, for the above firm and consumer, (1) implies
   $$\frac{dp_x}{dw} = \frac{p_x}{w} . \tag{2}$$

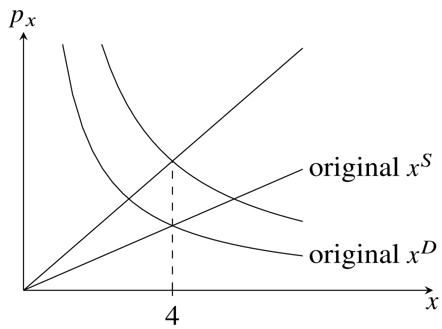

Figure 1. The market for good x. Demand curves are denoted by x^D and supply curves are denoted by x^S.

(e) Show that

$$\frac{d(p_x/w)}{dw} = 0$$

and use this fact to explain why Figure 1 illustrates what happens in the market for x when w is exogenously increased. You should label the two unlabeled curves of Figure 1 and provide an intuitive explanation of what is going on.

(f) If the original situation had $w = 1$ and $p_x = 2$, and the new situation has $w = 2$, then what are the new values for p_x, for x, and for ℓ?

2. **[17 points]**

(a) Suppose an economy consists of two consumers, Jones "j" and Smith "s," and two commodities, apples "a" and bananas "b." Suppose that the total amounts of apples and bananas in this economy are given by

$$a_j + a_s = 1 \quad \text{and}$$
$$b_j + b_s = 1$$

and the utility functions for Jones and Smith are

$$u_j(a_j, b_j) = a_j \cdot b_j$$
$$u_s(a_s, b_s) = a_s + \ln(b_s+1)\,.$$

Derive the equation or equations which I must have used to draw Figure 2 (this economy's "utility possibility frontier"), which

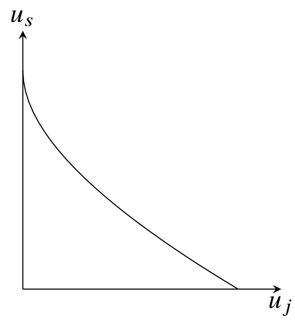

Figure 2. The utility possibility frontier.

shows all the possible utility levels for Jones and for Smith for all the Pareto Optimal allocations possible in this economy. Do this by:

i. eliminating a_s and b_s from your calculations;
ii. either giving the answer as one equation containing u_j and u_s, or giving the answer as two equations, each expressing u_j or u_s as a function of a_j (or b_j), and give the relevant upper and lower bounds of a_j (or b_j).

(I add these requirements because there are many equally-good ways of solving this problem and if students choose among these ways arbitrarily, it will take me a very long time to grade the varying answers.)

(b) What is the u_j-intercept and the u_s-intercept of the curve in Figure 2?

(c) If the initial endowments are $a_j = 0$, $b_j = 1$, $a_s = 1$, and $b_s = 0$, indicate in Figure 2 which part of the utility possibility frontier would be in the core of this economy.

3. **[18 points]** Imagine that there is a society composed of four members: Apu, Person 1 (P1), Person 2 (P2), and Person 3 (P3). Apu has supposedly committed a crime and as per the society's "constitution", after being presented with all the evidence, P1, P2 and P3 are now to vote on one of the three possible verdicts: A, L and D. The constitution allows Apu to choose the procedure by which the voting members P1, P2 and P3 reach a verdict. There are three different procedures (explained below) and all of them involve two stages of decision-making.

In *Stage 1* the voting members simultaneously and independently vote on a first issue. At the end of the first stage, the first-stage votes are made public (i.e. which issue each voting member voted for). In *Stage 2* the voting members again simultaneously and independently vote on a second issue. The final verdict is then delivered. In each stage the corresponding issue is decided by a majority voting rule meaning that an issue that receives two or more of all three votes in a stage is chosen. The three voting members are strategic/rational and each member's objective is to bring about an outcome which is best according to his/her ranking/preferences over A, L and D, which are as follows:

P1's ranking: $D \succ L \succ A$ where D is the best outcome, L is the middle and A is the worst outcome according to P1.

P2's ranking: $L \succ A \succ D$ where L is the best outcome, A is the middle and D is the worst outcome according to P2.

P3's ranking: $A \succ D \succ L$ where A is the best outcome, D is the middle and L is the worst outcome according to P3.

In all of the above cases, $x \succ y$, for example, indicates that x is strictly preferred to y. All of the above as well as the details of the following three procedures are common knowledge among all four members.

Procedure I: P1, P2 and P3 first vote on whether Apu is innocent (i) or guilty (g). If the majority vote for i in Stage 1, then Apu receives verdict A. Otherwise, in the second stage the members vote on which of the two remaining verdicts (D and L) should be applied to Apu.

Procedure II: P1, P2 and P3 first vote on whether Apu should receive verdict D. If the majority vote for Yes, then the verdict is D. If the majority vote for No, then in the second stage the members vote on whether Apu should receive verdict L. If the majority vote for Yes, then the verdict is L. If the majority vote for No, then the verdict is A.

Procedure III: P1, P2 and P3 first vote on which verdict is appropriate for Apu's crime (regardless of whether Apu committed the crime or not): L or D. In the second stage, they vote on whether Apu is innocent (i) or guilty (g). If the majority vote for i, then the verdict is A. If the majority vote for g, then the outcome is the verdict picked in the first-stage vote by the majority.

(a) Represent **Procedure I** as an extensive form game and find a subgame perfect Nash equilibrium of the extensive form game. (5 points)

(b) Represent **Procedure II** as an extensive form game and find a subgame perfect Nash equilibrium of the extensive form game. (5 points)

(c) Represent **Procedure III** as an extensive form game and find two subgame perfect Nash equilibria of the extensive form game. (7 points)

(d) Suppose Apu's preferences over the three possible verdicts are as follows: $A \succ L \succ D$ where A is the best outcome, L is the middle and D is the worst outcome according to Apu. Which of the three procedures will Apu ask the voting members to use to reach the final verdict? (1 point)

For parts (a), (b), and (c), the following three points apply: (1) you do not need to write the payoffs of P1, P2 and P3 at the terminal nodes, instead you can only write the voting outcome at the terminal nodes; (2) you can use—whenever possible—either the weakly dominant strategy equilibrium or the iterated elimination of weakly dominated strategies as the solution concept; and (3) for all three extensive form games, be sure to clearly label all player moves, players' actions and information sets to receive full credit.

4. **[18 points]** Apu is selling a nondurable good to Dipu. There are two periods, $t = 1, 2$, and Dipu can purchase one unit in each of the two periods, can purchase only one unit in either of the two periods, or may not purchase a unit at all in either of the two periods. Both Apu and Dipu discount future at the common rate $\delta \in (0, 1)$. Dipu's valuation for one unit of the product is v and it remains the same over both periods. Dipu's payoff/utility is given by: $d_1\,(v-p_1)+d_2\,\delta\,(v-p_2)$, where p_t is the price charged by Apu in period t, and $d_t = 1$ if Dipu purchases in period t and $d_t = 0$ otherwise. $p_1, p_2 \in R_+$ (that is, prices are non-negative). Apu's payoff/utility is given by: $d_1 p_1 + d_2 \delta p_2$. It is common knowledge that Apu's cost of production is zero. There is incomplete information about Dipu's valuation. Nature draws Dipu's valuation (type) according to the probability distribution that assigns probability 0.5 to $v = v_H$ and probability 0.5 to $v = v_L$.

The game proceeds as follows: In the first period (i.e., $t = 1$), Apu announces a price, p_1. Nature then draws Dipu's valuation (type) according to the probability distribution mentioned above which Dipu learns and then decides whether to buy or not to buy. In the second period (i.e., $t = 2$), Apu again announces a price, p_2, after learning whether Dipu had bought or not in the first period. Finally, Dipu decides whether to buy or not to buy. Assume that $0 < 2v_L < v_H$ and restrict attention to **pure strategies**.

(a) Write down the (extensive form) strategies of Apu and Dipu in the two-period game. Use clear mathematical notations to describe each player's strategies. [Hint: there should be two strategies for each player; one in $t = 1$ and another in $t = 2$.] (4 points)

(b) Consider a subgame starting at $p_1 \in [v_L, \delta v_L + (1 - \delta)v_H]$ (in the first period after Apu announces the price, but before Dipu learns his type and decides whether to buy or not). Find a separating perfect Bayesian equilibrium of the above subgame. [Hint: separating (or separation of) strategies should occur in the first period. More hint: how will Apu price in $t = 2$ if he assigns probability 1 to the value $v = v_H$? To the value $v = v_L$?] (7 points)

(c) Again consider a subgame starting at $p_1 \in (v_L, \delta v_L + (1-\delta)v_H]$ (in the first period after Apu announces the price, but before Dipu learns his type and decides whether to buy or not). Note now that $p_1 \neq v_L$. Is there a pooling perfect Bayesian equilibrium in these subgames? [Hint: how will Apu price in $t = 2$ if his beliefs equal his prior on v? Also remember that $0 < 2v_L < v_H$.] (7 points)

Section 2.
Answer one of the following two questions.

1. **[14 points]**

   Apu (henceforth A) and Dipu (henceforth D) each simultaneously chooses an effort level, $e_i \in [0, \infty)$ to help a third party, Pipu. The cost of effort for A is $c_A(e_A) = e_A^2/15$ and for D is $c_D(e_D) = e_D^2/10$. Pipu's satisfaction level is equal to the total effort that A and D exert, up to a maximum satisfaction level of 10. That is, Pipu's satisfaction level is equal to $\min\{e_A + e_D, 10\}$. Apu's utility/payoff is the difference between Pipu's satisfaction level and Apu's own effort cost. Similarly, Dipu's utility/payoff is the difference between Pipu's satisfaction level and Dipu's own effort cost. Apu and Dipu both want to maximize their individual payoff. Call the above strategic situation, game G_1, the details of which are common knowledge between A and D.

   (a) Write down the utility/payoff function of Apu and Dipu in G_1. (2 points)

   (b) Derive the best response function of Apu and the best response function of Dipu in G_1. (3 points)

   (c) Plot the best response functions of Apu and Dipu in G_1 in the same diagram in which the horizontal axis should represent e_A and the vertical axis should represent e_D. Use this diagram to identify all the pure strategy Nash equilibria of G_1. A diagram without proper labels will not receive full credit. (2 points)

   (d) Now consider a sequential version of the interaction between A and D, called game G_2, where first A chooses an effort level, and after observing A's choice, D chooses his own effort level. Find all the pure strategy subgame perfect Nash equilibria of G_2. (3 points)

   (e) In game G_3, the interaction between A and D is the same as in G_2 plus an additional stage of decision-making by A. After D chooses his own effort level (after observing A's choice), A observes D's choice and decides whether to exert some additional effort $\delta_A \in [0, \infty)$, so that his own total effort is $e_A + \delta_A$. Find all pure strategy subgame perfect Nash equilibria of G_3 when $e_A, e_D \in [0, 10]$ (instead of $[0, \infty)$) and $\delta_A \in [0, 10 - e_A]$ (instead of $[0, \infty)$). (4 points)

2. **[14 points]** Two toy-making firms (F1 and F2) play a two-stage game. In *Stage 1* both firms play a perfect and complete information game where first F1 decides the quality of its toy: g (good) or b (bad). After observing F1's choice, F2 also decides the quality of its toy: g (good) or b (bad). In *Stage 2* the qualities of the two toys become common knowledge and F1 and F2 play a simultaneous game where each toymaker chooses the price of its toy. There is a constant marginal cost of 2 for producing g-quality toys and the cost of producing a b-quality toy is zero. Fixed cost is zero for each firm. When both firms choose g, the market demand function for toys is given by $Q = 10 - P$ (where Q is the total quantity sold, P is the lowest price; consumers buy only from the lowest-price firm and, if both firms set the same price, then consumers split themselves equally between the two firms). When both firms choose b, the market demand function for toys is given by $Q = 4 - P$ (where Q is the total quantity sold, P is the lowest price; consumers buy only from the lowest-price firm and, if both firm set the same price, then consumers split themselves equally between the two firms). When one firm chooses g and the other chooses b, then the market demand functions are as follows: for the firm that chose g:

$$q_g = 10 - \frac{p_g}{4} + \frac{p_b}{4},$$

and for the firm that chose b:

$$q_b = \frac{p_g}{4} - \frac{p_b}{4},$$

where p_g is the price charged by the firm that chose g and p_b is the price charged by the firm that chose b. There is no discounting.

(a) Assume that each firm can only choose one of the following two prices: p_1 and p_2. Draw the extensive form of the above game (you do not need to write the players' payoffs at the terminal nodes). Label all player moves, players' actions and information sets to receive full credit. (2 points)

(b) Suppose p_1 and p_2, as described in part (a), each can be any non-negative number. Find the pure strategy subgame perfect Nash equilibrium of the above game with p_1 and p_2 each being any non-negative number. Note that now you will need to compute the players' payoffs and write down the players' payoffs at the terminal nodes. Write down the entire equilibrium strategy profile of each firm (i.e., what each firm does in equilibrium at each of its information sets) to receive full credit. (5 points)

Suppose now that the toy industry does not exist yet and the local administration decides to conduct an auction involving only two bidders and what is being auctioned is the right to be F1 in the above game. Thus whoever wins the auction will be F1 in the above two-stage game and whoever loses the auction will be F2. Call the participants in the auction Players A and B. In all of the auctions of Parts (c)–(e) the following applies: (1) the auction is a simultaneous sealed bid auction, (2) the winner of the auction is the player who submits the higher bid (the other player is called the loser), (3) if the two bids are the same, then Player A will be declared the winner (due to nepotism), and (4) bids can be any non-negative real number. All of this is common knowledge between the two bidders. There is no discounting.

With the above additional stage, the firms now play a three-stage game. In the first stage, the auction takes place. In the second stage, firms choose qualities (as described above) after the outcome from the auction stage is known to both the firms. In the third stage, firms choose prices (as described above) after the outcome from the quality choice stage is known to both the firms.

(c) The auction is a first-price auction (the winner pays her own bid and the loser pays nothing). Find all the pure strategy subgame perfect Nash equilibria of the three-stage game just described above. An answer that correctly identifies the pure strategy subgame perfect Nash equilibria without a clear explanation will not receive full credit. (2 points)

(d) The auction is a second-price auction (the winner pays the bid of the loser and the loser pays nothing). Find all the pure strategy subgame perfect Nash equilibria of the three-stage game just described above. An answer that correctly identifies the pure strategy subgame perfect Nash equilibria without a clear explanation will not receive full credit. (3 points)

(e) The auction is an all pay first-price auction (each player pays her own bid, including the loser). Find all the pure strategy subgame perfect Nash equilibria of the three-stage game just described above. An answer that correctly identifies the pure strategy subgame perfect Nash equilibria without a clear explanation will not receive full credit. (2 points)

Section 3.
Answer one of the following two questions.

1. **[16 points]** Suppose a consumer consumes two goods, x and y, has income m, and has the "Constant Elasticity of Substitution" utility function $u(x, y) = (x^{1/2} + y^{1/2})^2$. Suppose

   - the price of y, denoted p_y, is always equal to one.

   You may use without proof the facts that Varian's book states on its p. 112, namely that:

   - if a consumer has the CES utility function $u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}$
   - and if we define $r = \frac{\rho}{\rho - 1}$
   - then the consumer's expenditure function is $e(\mathbf{p}, u) = (p_1^r + p_2^r)^{1/r}\, u$,
   - their indirect utility function is $v(\mathbf{p}, m) = (p_1^r + p_2^r)^{-1/r}\, m$,
   - their demand curve for x_1 is $x_1(\mathbf{p}, m) = \dfrac{m p_1^{r-1}}{p_1^r + p_2^r}$,
   - and their money metric utility function is $\mu(\mathbf{p}; \mathbf{q}, m) = (p_1^r + p_2^r)^{1/r} (q_1^r + q_2^r)^{-1/r} m$.

   On p. 161 of Varian's book, upon defining

   $$\mu(\mathbf{q}; \mathbf{p}, m) = e(\mathbf{q}, v(\mathbf{p}, m)),$$

   Varian gives the equivalent and compensating variation as, respectively,

   $$EV = \mu(\mathbf{p}^0; \mathbf{p}', m') - m^0 \quad \text{and}$$
   $$CV = m' - \mu(\mathbf{p}'; \mathbf{p}^0, m^0).$$

   (a) The consumer's "expenditure on x" is the price of x times the quantity of x which he buys, in other words, $x p_x$. Assuming that $p_x = 3$, find an expression for this consumer's expenditures on x. (This expression will depend on m.)

   (b) Assuming that $p_x = 3$, find an expression for this person's consumer surplus generated from x. (This expression will depend on m.) You may use without proof the following result:

   $$\int \frac{p^{-2}}{p^{-1} + 1}\, dp = \int \frac{dp}{p + p^2} = \int \frac{1 + p - p}{p\,(1 + p)}\, dp$$

$$= \int \Big[\frac{1+p}{p\,(1+p)} - \frac{p}{p\,(1+p)}\Big]\, dp = \int \Big(\frac{1}{p} - \frac{1}{1+p}\Big)\, dp$$

$$= \ln p - \ln(1{+}p)\,.$$

(c) Express EV and CV in terms of the expenditure function [better: in terms of income and the old and new prices] and describe the relationship between EV and CV, on the one hand, and "willingness to pay" ("WTP") and "willingness to accept" ("WTA"), on the other hand.

(d) Find this consumer's WTP for a decrease in the price of x from infinity to 3. Assume as before that $p_y = 1$. (Your expression for WTP will depend on m.)

(e) Find this consumer's WTA for an increase in the price of x from 3 to infinity. Assume as before that $p_y = 1$. (Your expression for WTA will depend on m.)

(f) Sketch a graph showing how this person's expenditures on x, consumer surplus generated from x, WTP, and WTA all depend on m. Use your results from (a), (b), (d), and (e) to do this.

2. **[16 points]** Suppose a consumer consumes two goods, x and y, has income m, and has the "Constant Elasticity of Substitution" utility function $u(x, y) = (x^{1/2} + y^{1/2})^2$. Suppose

- the price of y, denoted p_y, is always equal to one;
- and the consumer's income $m = 100$.

You may use without proof the facts that Varian's book states on its p. 112, namely that:

- if a consumer has the CES utility function $u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}$
- and if we define $r = \frac{\rho}{\rho - 1}$
- then the consumer's expenditure function is $e(\mathbf{p}, u) = (p_1^r + p_2^r)^{1/r}\, u$,
- their indirect utility function is $v(\mathbf{p}, m) = (p_1^r + p_2^r)^{-1/r}\, m$,
- their demand curve for x_1 is $x_1(\mathbf{p}, m) = \dfrac{m p_1^{r-1}}{p_1^r + p_2^r}$,
- and their money metric utility function is $\mu(\mathbf{p}; \mathbf{q}, m) = (p_1^r + p_2^r)^{1/r} (q_1^r + q_2^r)^{-1/r} m$.

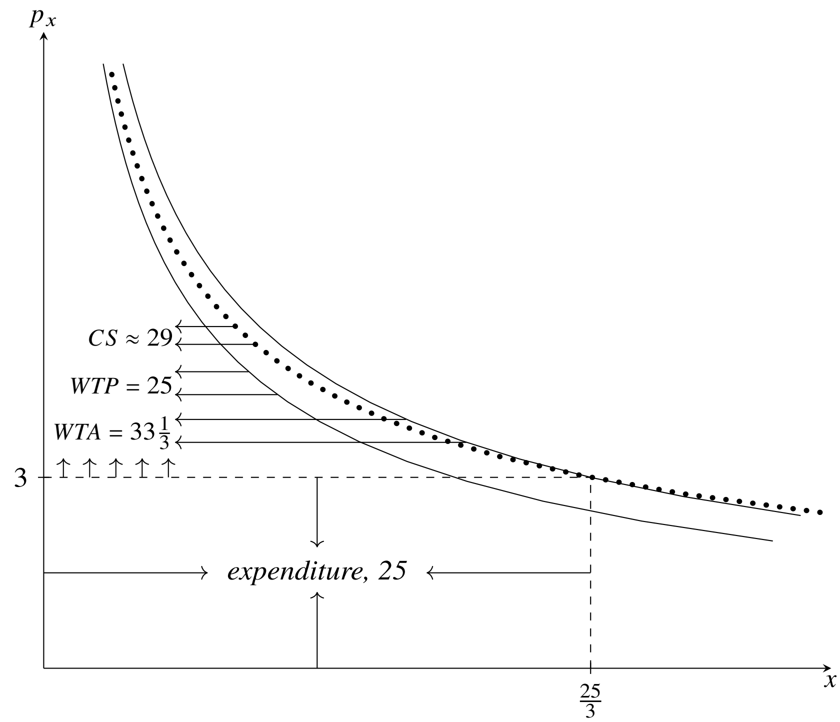

Figure 3. Demand curves for an income of $m = 100$ and a utility function of $u(x, y) = (x^{1/2} + y^{1/2})^2$ when $p_y = 1$.

On p. 161 of Varian's book, upon defining

$$\mu(\mathbf{q}; \mathbf{p}, m) = e(\mathbf{q}, v(\mathbf{p}, m)) ,$$

Varian gives the equivalent and compensating variation as, respectively,

$$EV = \mu(\mathbf{p}^0; \mathbf{p}', m') - m^0 \quad \text{and}$$
$$CV = m' - \mu(\mathbf{p}'; \mathbf{p}^0, m^0) .$$

If there are other results which Varian proves which you want to use, simply cite the result and its page number; since this is an open-book exam, I think it's pointless to ask you to copy a proof from Varian's book straight onto your exam paper.

Please look at Figure 3.

(a) The "$WTA = 33\frac{1}{3}$" label of Figure 3 denotes willingness to accept the price of x increasing from three to infinity, and $33\frac{1}{3}$ is the area to the left of the right-most curve and above the line $p_x = 3$. Show that the equation of this right-most curve is

$$\frac{u}{(1+p_x)^2}$$

where

$$u = \left(\sqrt{\frac{25}{3}} + \sqrt{75}\right)^2 = 133\tfrac{1}{3}\,.$$

(b) The "$WTP = 25$" label of Figure 3 denotes willingness to pay in return for the price of x decreasing from infinity to three, and 25 is the area to the left of the left-most curve and above the line $p_x = 3$. Show that the equation of this left-most curve is

$$\frac{u}{(1+p_x)^2}$$

where

$$u = \left(\sqrt{0} + \sqrt{100}\right)^2 = 100\,.$$

(c) The "$CS \approx 29$" label of Figure 3 denotes consumer surplus, and $100 * \ln(4/3) \approx 29$ is the area to the left of the dotted curve and above the line $p_x = 3$. What is the equation of this dotted curve? (You may express this either as a function of p_x or as a function of x.)

(d) Consider the following two situations:

   i. The situation depicted in Figure 3, with $p_x = 3$ and $p_y = 1$ and $m = 100$ and consumer surplus from consumption of x approximately equal to 29 and consumer surplus from consumption of y (not illustrated); or

   ii. A situation in which the consumer faces a price-discriminating seller who charges the consumer \$25 + \$29 for consuming $x = 25/3$ (which the consumer does consume and does pay) and the consumer faces a uniform price of $p_y = 1$ for y and the consumer is given an income of \$100 plus \$34.

In which situation does this consumer have the larger consumer surplus (considering both goods x and y)? There is no need to provide a numerically-calculated answer; one arrived at just by logical reasoning is what is being asked for. (This question may be too easy but it becomes interesting in light of the next question.)

(e) In which of the situations described in part (d) does this consumer have the larger utility? There is no need to provide a numerically-calculated answer; one arrived at just by logical reasoning is what is being asked for.