There are 100 points possible on this exam, 50 points each for Prof. Lozada's questions and Prof. Dugar's questions.

There are three sections on this exam:

- In the first section contains all of the required questions. There are four of them. The first two are worth 17 points each; the second two are worth 18 points each.
- In the second section there are two questions; you should work one of them. Each is worth 14 points.
- In the third section there are two questions; you should work one of them. Each is worth 16 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

Answers with illegible or difficult-to-read-handwriting may lower your grade because we may not be able to read and understand your answers, especially considering that we are looking at photocopies. So it is in your best interest to make your answers readable.

It would be helpful for you to put the number of the problem you are working on at the top of every page of your answers, so we do not accidentally ignore part of your answer.

In this document, v and w respectively denote the Roman lowercase "v" and "w." Also, in this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

Good luck.

Section 1.
Answer all of the following four questions.

1. **[17 points]** Consider a cost-minimizing firm which produces an output y using two inputs x_1 and x_2 according to the production function $y = f(x_1, x_2)$. Make the usual assumptions that the firm takes the prices of x_1 and x_2 to be fixed, that both $\partial f / \partial x_1$ and $\partial f / \partial x_2$ are strictly positive, and that there are diminishing returns to each input. Also assume that the second-order conditions for cost minimization are satisfied.

   Call an input "inferior" if when output increases, the firm chooses to use less of this input.

   (a) Is it possible for both x_1 and x_2 to be inferior (simultaneously)? You should be able to answer this without solving any optimization problem; indeed, undergraduate students who do not know calculus should be able to answer this.

   (b) Under what conditions is x_1 inferior? (This is not a question undergraduates could solve.)

   (c) Under what conditions is x_2 inferior?

   (d) Use the answers to (a), (b), and (c) to argue that it is impossible for f''_{12} to be very negative.

   (e) Could x_1 or x_2 ever actually be inferior? When?

2. **[17 points]** Consider a two-person, two-commodity economy in which "x_{ij}" represents the amount of commodity i belonging to person j. Suppose the utility function of person 1 is

$$\ln x_{11} + \ln x_{21}$$

   and the utility function of person 2 is

$$\ln x_{12} + \ln x_{22} .$$

   Suppose the initial endowments of persons 1 and 2 are $\omega_1 = (1, 1)$ and $\omega_2 = (2, 1)$, respectively. Find the core of this economy.

3. **[18 points]** Apu is considering selling a product to a single buyer, Dipu, who has constant marginal utility θ for Apu's product. Specifically, if Dipu has a marginal utility of θ and buys q units of the product by paying a total revenue TR, Dipu's net utility is: $u(q, TR, \theta) = \theta q - TR$. We assume that Dipu's outside option is 0.

   Apu has a cost function $c(q) = 0.5cq^2$. Apu's total profit, if he sells q units at a total revenue TR, is: $\pi(TR, q) = TR - 0.5cq^2$.

(a) Suppose Apu knows that Dipu's marginal utility is θ. Derive, under this full information condition, both as a function of θ, the quantity $q(\theta)$ and total revenue $TR(\theta)$ that Apu offers to Dipu to maximize his profit. *(3 points)*

(b) Suppose now that Apu does not know Dipu's marginal utility, and instead Apu believes that Dipu's marginal utility is drawn from the uniform distribution over $[0, 1]$. Now, Apu decides to offer Dipu a menu of two bundles: a bundle (q_1, TR_1) (call it "bundle 1") intended for Dipu if his type $\theta \in [\theta_1, \theta_2)$, and a bundle (q_2, TR_2) (call it "bundle 2") if his type is $\in [\theta_2, 1]$. Assume that $0 < \theta_1 < \theta_2 < 1$. The values of θ_1 and θ_2 are fixed.

Write down Apu's expected profit maximization equation when Apu would like $\theta \in [\theta_1, \theta_2)$ type to purchase bundle 1, $\theta \in [\theta_2, 1]$ type to purchase bundle 2, and $\theta \in [0, \theta_1)$ type to purchase nothing. *(4 points)*

(c) Write down all the incentive compatibility constraints and participation or individual rationality constraints for Apu's profit maximization problem. *(5 points)*

(d) Solve for Apu's q_1, q_2, and total revenues TR_1 and TR_2, given the values of θ_1 and θ_2. You may assume that $\theta_1 + \theta_2 > 1$. Write down all the steps to receive all the points. *(6 points)*

4. **[18 points]** Apu is a homeowner with the expected utility function $u(x) = 1 - e^{-x}$, where x is the wealth level measured in million USD and $u(x)$ satisfies the expected utility hypothesis. Apu's entire wealth is his house and the value of his house is 1 (million USD). However, his house can be destroyed by a cyclone that will reduce its value to 0. The probability of a cyclone destroying his house and reducing its value to 0 is given by $\pi \in (0, 1)$.

(a) What is the largest premium P that Apu is willing to pay for full insurance? (Apu pays the premium P and gets back 1 in case of a cyclone, making his wealth $1 - P$ regardless of the cyclone.) *(4 points)*

(b) Suppose a local insurance company, *Inslocal*, has insured n identical houses, all in the neighborhood of Apu, for a premium of P per house. Suppose also that with probability π there can be a cyclone in the neighborhood destroying all houses (i.e., either all houses are destroyed or none of them is destroyed). Suppose finally that P is small enough that Apu has insured his house. Having insured his house, what is the largest Q that Apu is willing to pay to get a $1/n$ share of the company? *Your answer should be a number for Q.* (The value of *Inslocal* is the total premium it collects minus the payments to the insured homeowners in case of a cyclone.) *(5 points)*

(c) Answer part (b) assuming now that the insurance company is called *Insglobal* and it maintains global operations. It insured n identical houses in different parts of the world (all outside of Apu's neighborhood), so that the destruction of houses by cyclone are all independent (i.e., the probability of a cyclone in one house is π independent of how many other houses has been destroyed by a cyclone). *Your answer should be a mathematical expression for Q. Discuss briefly why your answers in part (b) and part (c) differ. (7 points)*

(d) *For this part* assume that n is large enough so that

$$\sum_{k=0}^{n} C_{n,k}\, e^{k/n} \pi^k (1-\pi)^{n-k} \cong e^{\frac{\pi+\pi(1-\pi)}{2n}}$$

where "$\cong$" denotes "is approximately equal to," $C_{n,k}$ denotes the number of k combinations out of n and where the sum is one minus the expected payoff from the loss due to the payments to the cyclone-affected houses. *What is the economic interpretation of the final expression that you get after using the mathematical relationship given in this part with your answer in part (c)? (2 points)*

Section 2.
Answer one of the following two questions.

1. **[14 points]** Apu is a graduate from the "U" currently searching for a job. Apu has received a job offer from Company B that pays him wage r. In this question, Company B is, however, not considered as a player. Instead, Apu and Company A are the two players in this game and Apu and Company A are currently negotiating. They use alternating offer bargaining, Apu offering at even dates $t = 0, 2, 4, \ldots$ and Company A offering at odd dates $t = 1, 3, 5, \ldots$. When Apu makes an offer w, Company A either accepts the offer by hiring Apu at wage w and ends the negotiation, or rejects the offer and the negotiation continues. When Company A makes an offer w, Apu

   - either accepts the offer w and starts working for Company A for wage w, and thus ending the game, or
   - rejects the offer w and takes Company B's offer r, working for Company B for wage r and thus ending the game, or
   - rejects the offer w and then the negotiation continues.

   If the game continues to date $\bar{t} \leq \infty$, then the game ends with zero payoffs for both players. If Apu takes Company B's offer at $t < \bar{t}$, then the payoff of Apu is $r\,\delta^t$ and the payoff of Company A is 0, where $\delta \in (0, 1)$. If Apu starts working for Company A at $t < \bar{t}$ for wage w, then Apu's payoff is $w\,\delta^t$ and Company A's payoff is $(\pi - w)\,\delta^t$, where $\pi/2 < r < \pi$. (Note that Apu cannot work for both A and B.)

   (a) Compute the subgame-perfect equilibrium for $\bar{t} = 4$. (There are four rounds of bargaining.) Show all your work to receive all the points. *(6 points)*

   (b) Take $\bar{t} = \infty$. Conjecture a subgame-perfect equilibrium and check that the conjectured strategy profile is indeed a subgame-perfect equilibrium. Show all your work to receive all the points. *(8 points)*

2. **[14 points]** Two children play a signaling game in which they are fighting for an apple (which they found on their way back from school) which is worth v to each. Child 1 can be of two types: strong or weak. The strong type occurs with probability p and the weak type occurs with probability $(1-p)$. Child 1's type is his private information and is unknown to Child 2. However, Child 2 knows the probability distribution of the two

types. For the sake of simplicity, assume that Child 2 is of some known strength. Also assume that both the children are risk neutral.

The game proceeds as follows: In period 1, Child 1 *jumps* and Child 2 watches. In period 2, Child 2 decides whether to engage in *fight* or *flight*. Child 1 never leaves (i.e., never plays *flight*).

In period 1, Child 2 observes how high Child 1 jumps. If Child 1 jumps to a height of h, the energy cost to Child 1 is $e_s(h)$ if he is strong and $e_w(h)$ if he is weak. For all $h > 0$ the energy cost functions satisfy the following conditions:

- $e_s(0) = 0$,
- $e_s'(h) > 0$,
- $e_s''(h) > 0$;
- $e_w(0) = 0$,
- $e_w'(h) > 0$,
- $e_w''(h) > 0$;
- $e_s(h) < e_w(h)$,
- $e_s'(h) < e_w'(h)$.

In period 2, a fight costs each child f. A strong Child 1 wins the fight against Child 2 with probability λ_s, and a weak Child 1 wins the fight against Child 2 with probability λ_w, with $\lambda_s > \lambda_w$. The winner enjoys the apple, and the loser gets nothing. If Child 2 leaves (i.e., plays *flight*), then Child 1 enjoys the apple without incurring f. Assume that $(1-\lambda_w)\, v > f$, $(1 - \lambda_s)\, v < f$, and $[\,p\,(1 - \lambda_s) + (1 - p)\,(1 - \lambda_w)]\, v < f$.

(a) Represent the above game in extensive form. Clearly label the extensive form game to receive all the points. *(4 points)*

(b) Find the conditions on payoffs for all separating and pooling equilibria in this game. Show all your work to receive all the points. *(10 points)*

Section 3.
Answer one of the following two questions.

1. **[16 points]**

   [Completely optional introduction: This is the beginning and the middle but not the end of a demonstration that George Stigler's "Coase Theorem" is false in the general case of goods having arbitrary income effects. (This would please Nobel Laureate Ronald Coase but it profoundly challenges followers of Stigler.)]

   (a) Suppose a consumer's welfare depends on the number of apples a which he consumes and on the amount of clean air in his environment. There is a polluting firm in the consumer's environment and the amount of air pollution it emits is proportional to the level of its output Q. For some fixed level of output $\overline{Q} > 0$, argue that

   $$u(a, Q) = a \cdot (\overline{Q} - Q)$$

   is a reasonable specification for this consumer's utility function.

   (b) Suppose this consumer sets out one day with m dollars to visit the marketplace and buy some apples. Before he gets to the marketplace, he encounters the owner of the polluting firm. He may strike up a conversation with this owner in the hopes of affecting how much the firm pollutes. Perhaps he and the firm owner exchange money for a change in Q. Let m_a denote the amount of money the consumer has when he takes leave of the firm owner and proceeds to the marketplace, at which time the amount of Q, and therefore air pollution, is irrevocably fixed (it will never change again).

   Show that his utility at this point is destined to be

   $$\frac{m_a}{p_a} (\overline{Q} - Q)$$

   where p_a is the price of apples.

   (c) Suppose that in this country, firms have the right to emit pollution at will. (One could say that the firm has the "property right" to pollute.) Suppose that in the absence of any interaction or bargaining between the firm and the consumer,

   the firm sees fit to produce $\overline{Q}/2$ units of output.

Show that with that level of output, the (indirect) utility of the customer in this initial situation would be

$$v_0 = \frac{m\overline{Q}}{2p_a}\,.$$

(d) Upon meeting the firm owner, the consumer contemplates offering the firm owner money in return for a reduction of Q. If the consumer offered the firm owner T dollars and in return the firm owner reduced output to Q, show that the consumer would, after making the bargain and then buying apples, have a utility level of

$$v' = \frac{m - T}{p_a}\,(\overline{Q} - Q)\,.$$

(e) Suppose that, for a given Q, the consumer is indifferent between paying $T(Q)$ in return for the firm producing only Q, on the one hand, and paying nothing and having the firm produce $\overline{Q}/2$, on the other hand. Find T as a function of Q.
Hint: I get

$$T = m\,\frac{\overline{Q} - 2Q}{2\overline{Q} - 2Q} > 0 \quad \text{for } Q < \overline{Q}/2.\,.$$

(f) Show that

$$\frac{dT}{dQ} = \frac{-m\overline{Q}}{2\,(Q - \overline{Q})^2} < 0 \quad \text{for } Q < \overline{Q}/2, \text{ and that}$$
$$\frac{d^2T}{dQ^2} = \frac{m\overline{Q}}{(Q - \overline{Q})^3} < 0 \quad \text{for } Q < \overline{Q}/2.$$

(g) Make a rough sketch of $T(Q)$, indicating the values of $T(0)$ and of $T(\overline{Q}/2)$.

(h) If EC denotes the "external cost" which pollution imposes on this consumer, argue that

$$EC(Q) = T(0) - T(Q)\,.$$

(i) Show that the "marginal external cost"

$$MEC = \frac{d\,EC}{dQ} = \frac{m\overline{Q}}{2\,(Q - \overline{Q})^2} > 0\,.$$

(Prove the second equality.) Also show that

$$\frac{d\,MEC}{dQ} = \frac{-m\overline{Q}}{(Q-\overline{Q})^3} = \frac{m\overline{Q}}{(\overline{Q}-Q)^3} > 0\,.$$

(Prove at least one of the equalities and prove the inequality.)

(j) Now we contrast this situation to that under a different constitution in which consumers have the right to clean air and firms cannot pollute the air without obtaining permission from the consumer. (One could say that consumers have the "property right" to clean air.) Show that in the absence of any interaction or bargaining between the firm and the consumer, (indirect) utility of the customer in this initial situation would be

$$v_0 = \frac{m\overline{Q}}{p_a}\,.$$

(k) Upon meeting the firm owner, the consumer contemplates offering to allow the firm to increase output to Q in return for the firm paying the consumer $\widehat{T}$ dollars. Show that the consumer would, after making the bargain and then buying apples, have a utility level of

$$v' = \frac{m+\widehat{T}}{p_a}\,(\overline{Q}-Q)\,.$$

(l) Suppose that, for a given Q, the consumer is indifferent between receiving $\widehat{T}(Q)$ in return for allowing the firm to increase its production to Q, on the one hand, and receiving nothing and making no bargain with the firm, on the other hand. Show that

$$\widehat{T} = \frac{mQ}{\overline{Q}-Q} > 0$$

and show that

$$\frac{d\widehat{T}}{dQ} = \frac{m\overline{Q}}{(\overline{Q}-Q)^2} > 0 \quad \text{and that}$$
$$\frac{d^2\widehat{T}}{dQ^2} = \frac{2m\overline{Q}}{(\overline{Q}-Q)^3} > 0\,.$$

(m) In this situation argue that external cost $\widehat{EC}(Q) = \widehat{T}(Q)$.

(n) Show that

$$\widehat{MEC} = \frac{d\,\widehat{MEC}}{dQ} = 2\,MEC \quad \text{and} \quad \frac{d\,\widehat{MEC}}{dQ} = 2\,\frac{d\,MEC}{dQ}\,.$$

2. **[16 points]**

[Completely optional introduction: This will show that it is incorrect to use the change in "consumer surplus" as a measure of welfare change in the general case of goods having arbitrary income effects.]

(a) Suppose a consumer has a utility function $u = x_1^{1/2} x_2^{1/2}$ and income $m = 2$ and takes the prices p_1 and p_2 as given. If x_1 is "cheese," find the consumer's (Marshallian) demand curve for cheese.

(b) Make a rough, somewhat large sketch of this consumer's demand curve for cheese for $0 < x_1 = 1$ and identify the quantity demanded of cheese for prices p_1 of 1, 2, 3, and 4 dollars per pound ("\$/lb") of cheese.

(c) Consider the following explanation of consumer surplus, which resembles what one might find in an undergraduate microeconomics textbook.

> Consumer surplus, which is the area under the demand curve, measures how much a consumer would be willing and able to spend to buy cheese. To illustrate this, consider how much money the consumer whose demand curve you drew in part (b) would be willing and able to spend to buy a certain total amount of cheese. If the price of cheese were \$4/lb, he would be willing to buy *[fill in this blank, which is part (i) of this sub-part]* pounds of cheese, and so would spend the amount of money shown by area *[fill in this blank, which is part (ii) of this sub-part]* in the diagram. *[Designate areas in your graph by giving labels such as A, B, C, etc. to the vertices of those geometric areas, rather than say by shading the areas, because shading may make part (d) harder to superimpose onto this graph.]*
>
> If after making this transaction the price of cheese were to fall to \$3/lb, he would be willing to buy more cheese, raising his total cheese purchases to *[fill in this blank, which is part (iii) of this sub-part]* pounds of cheese, and so would in total spend the amount of money shown by area *[fill in*

this blank, which is part (iv) of this sub-part] in the diagram.

If after making this transaction the price of cheese were to fall further, to \$2/lb, he would be willing to buy more cheese, raising his total cheese purchases to *[fill in this blank, which is part (v) of this sub-part]* pounds of cheese, and so would in total spend the amount of money shown by area *[fill in this blank, which is part (vi) of this sub-part]* in the diagram.

If, finally, after making this transaction the price of cheese were to fall even further, to \$1/lb, he would be willing to buy more cheese, raising his total cheese purchases to *[fill in this blank, which is part (vii) of this sub-part]* pounds of cheese, and so would in total spend the amount of money shown by area *[fill in this blank, which is part (viii) of this sub-part]* in the diagram. This amount of money is approximately equal to consumer surplus and thus shows that *[fill in this blank with the conclusion of this argument, which is part (ix) of this sub-part]*.

(d) In this part you have to show that the explanation in part (c) is wrong. To do this, suppose the consumer has already spent the money to purchase, at a price of \$4/lb, the amount of cheese you answered in sub-part (i) of part (c). Suppose the consumer has taken ownership of this amount of cheese but has not eaten it yet. Before eating this cheese and before buying any x_2, the consumer gets the opportunity to buy more cheese at a price of \$3/lb.

  i. Show that he will not buy the total amount of cheese given in sub-part (iii) of part (c) by showing that the total amount of cheese he will actually buy is $7/24 \approx 0.29$ (where "$\approx$" means "is approximately equal to"). Hint: first calculate how much *extra* cheese he will buy.

  ii. Superimpose onto your prior graph this consumer's new demand curve for cheese for prices of 3, 2, and 1 dollars per pound, giving a numerical value for the amount of cheese demanded at each of these prices.

  iii. Construct an argument that the consumer surplus described in part (c) is not actually "how much a consumer would be willing and able to spend to buy cheese." Include a conceptual explanation of why the the demand curve you derived in part (a) generated a misleading answer to part (c).