Sample Math Exam for Entering Ph.D. Students
Dept. of Economics, University of Utah

This exam has 66 points. There are 11 questions on the exam; each is worth 6 points. You should work all of the questions.

You have **90 minutes** to finish this test, which gives you approximately 8 minutes per question. Answer the questions using as much precision and detail as the time allows, showing all of your work. Correct answers which are completely unsupported by explanations will not be awarded points.

Answer all of the following questions.

1. Consider the problem $\min x^2 + y^2$ subject to $x + 2y = a$. If the first-order conditions for this problem are satisfied at a particular point, would the second-order conditions be satisfied at that point? Why or why not?

2. Decide which of the following sets are convex:

   (a) $\{(x, y) : x^2 + y^2 > 8\}$

   (b) $\{(x, y) : x \geq 0,\ y \geq 0,\ xy \geq 1\}$.

3. The equation

   $$x^y + z^x = k$$

   where k is a positive constant, defines z as a function of x and y, for $x > 0$ and $y > 0$. Find the partial derivative of z with respect to x.

4. Let $F(K, L) = 10K^{1/2}L^{1/3}$ for $K \geq 0$ and $L \geq 0$. Find a constant a such that $F(tK, tL) = t^a F(K, L)$ for all $t > 0$, $K \geq 0$, and $L \geq 0$.

5. Find the eigenvalues and eigenvectors of the following:

   $$\begin{pmatrix} 1 & 4 \\ 6 & -1 \end{pmatrix}.$$

6. Using elementary row operations, find the inverse of

   $$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

7. Express the vector $(4, -11)$ as a linear combination of $(2, -1)$ and $(1, 4)$.

8. Decide where the function

   $$g(x) = \frac{1 - x}{1 + x}$$

   is convex.

9. Find the error in the following:

$$\lim_{x\to 1} \frac{x^2+3x-4}{2x^2-2x} = \lim_{x\to 1} \frac{2x+3}{4x-2} = \lim_{x\to 1} \frac{2}{4} = \frac{1}{2}\,.$$

What is the value of the first limit?

10. Find a quadratic approximation to $F(K) = AK^a$ about the point $K_0 = 1$.

11. According to the 20th report of the International Commission on Whaling, the number N of fin whales in the Antarctic for the period 1958–1963 was given by

$$N = -17,400\,t + 151,000$$

where $0 \le t \le 5$ and where $t = 0$ corresponds to January 1958, $t = 1$ corresponds to January 1959, and so on. If the decrease continued at the same rate, when would there be no fin whales left? If you do not have a calculator with you, modify the relationship to

$$N = -20,000\,t + 160,000\,.$$

End of Exam