There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are weighted differently from Prof. Kiefer's questions: Prof. Lozada's questions are worth 14 points, 14 points, and 8 points, while Prof. Kiefer's questions are worth 18 points, 9 points, and 9 points.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 14 points; the second is worth 14 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 8 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It is helpful (but not required) for you to put the number of the problem you are working on at the top of every page of your answers.

In this document,

$$v, \quad w, \quad \omega, \quad \boldsymbol{\omega}$$

denote, in the order in which they appear, the Roman lower-case "v" and "w" and the Greek lower-case "omega" and "boldface omega." Also, in this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

Good luck.

Section 1.
Answer all of the following three questions.

1. **[14 points]** Suppose a price-taking firm uses inputs x_1 and x_2 to produce output y according to the production function $y = \sqrt{x_1} + \sqrt{x_2}$. Find this firm's cost function.

   Explicitly verify that the second-order conditions hold in this particular problem. (It is not enough to state the conditions symbolically in terms of the derivatives of the Lagrangian.)

2. **[14 points]**

   (a) Answer the following questions assuming a competitive cost-minimizing firm.

   i. Prove that input demand curves $\mathbf{x}(\mathbf{w}, y)$ are homogeneous of degree zero in $\mathbf{w}$.

   ii. As some of you may already know, Euler proved the following: if $f(\mathbf{x})$ is differentiable and is homogeneous of degree k, then

   $$\nabla_{\mathbf{x}} f(\mathbf{x}) \cdot \mathbf{x} = k f(\mathbf{x}).$$

   (Do not forget that the left-hand side has a "$\cdot\, \mathbf{x}$" in it.) What property of input demand curves can you derive from this result, given what you already know from part (i)?

   iii. Rewrite your answer to part (ii) for the special case when the total number of inputs is exactly three.

   iv. For any two inputs j and k, here are two definitions:

   $$\partial x_j(\mathbf{w}, u)/\partial w_k \geq 0 \iff j \text{ and } k \text{ are "substitutes"}$$
   $$\partial x_j(\mathbf{w}, u)/\partial w_k < 0 \iff j \text{ and } k \text{ are "complements."}$$

   Use the previous parts of this question, and other information, to prove that if the total number of inputs is three, then every input has at least one substitute.

   v. Prove that every input has at least one substitute (regardless of what the total number of commodities may be).

   (b) Answer the following questions assuming a competitive profit-maximizing firm.

   i. Prove that net output curves $\mathbf{y}(\mathbf{p})$ are homogeneous of degree zero in $\mathbf{p}$.

ii. What property of net output curves can you derive from the result that if $f(\mathbf{x})$ is differentiable and is homogeneous of degree k, then $\nabla_{\mathbf{x}} f(\mathbf{x}) \cdot \mathbf{x} = k f(\mathbf{x})$, given what you already know from the part (i)?

iii. Rewrite your answer to the part (ii) for the special case when $\mathbf{y} \in \mathbf{R}^3$.

iv. For the special case when $\mathbf{y} \in \mathbf{R}^3$, tell me everything you know about the signs of

$$\frac{\partial y_1}{\partial p_1}, \quad \frac{\partial y_1}{\partial p_2}, \quad \text{and} \quad \frac{\partial y_1}{\partial p_3}$$

and thoroughly describe what these results mean intuitively.

3. **[18 points]**
Imagine a 2 by 2 economy with two consumers, $i = 1$ (Robinson) and 2 (Friday); each consumes two goods, leisure x_{1i} and fish x_{2i}. Their preferences are identical,

$$u(x_{1i}, x_{2i}) = x_{1i} - \frac{(x_{2i} - 3)^2}{2}.$$

Their endowments $\boldsymbol{\omega}_i = (\omega_{1i}, \omega_{2i}) = (4, 0)$ are also identical.

Fish can be produced according to the production function $y_2 = |y_1|$. General equilibrium is described by $x_{1i} = \omega_{1i} + y_{1i}$, $y_{11} + y_{12} = y_1$ and $x_{21} + x_{22} = y_2$. Define the price of leisure as 1.

(a) In perfect competition what is equilibrium price of a fish and the allocation $(x_{11}, x_{21}, x_{12}, x_{22})$?

(b) Now consider a pure monopoly in the fish market, and suppose that Robinson owns the fishing firm. Now what is price of a fish and the allocation?

(c) Consider a reform of the monopoly regime (b) in favor of perfect competition (a). Is this a Pareto improvement?

(d) Consider the allocation $(x_{11}, x_{21}, x_{12}, x_{22}) = (3, 2, 1, 2)$. Show that this is the competitive allocation given a redistribution of endowments as $\boldsymbol{\omega}_1 = (5, 0)$ and $\boldsymbol{\omega}_2 = (3, 0)$. Is it also a Pareto improvement on the monopoly allocation? Find the set of all Pareto efficient allocations. Illustrate your answer in utility (u^f, u^r) space.

(e) Which equilibrium would a Benthamite social planner prefer?

(f) Discuss the wider implications of this example.

Section 2.
Answer one of the following two questions.

1. **[8 points]**

   (a) A consumer consumes n commodities labeled 1, 2, 3, …, n. This consumer takes the prices of commodities 2, 3, 4, …, n as given, but this consumer can affect the price he pays for commodity 1 because the more of commodity 1 he buys, the lower the price he has to pay for commodity 1.

   For what commodities i, if any, is it true that this consumer's commodity demands obey

   $$x_i = -\frac{\partial v/\partial p_i}{\partial v/\partial m} \,?$$

   The letter v denotes the indirect utility function.

   (b) A firm has market power over commodities 1, 2, 3, …, m, but this firm has no market power over commodities m+1, m+2, …, n. Suppose that $m < n$.

   For what commodities i and j, if any, is it true that this firm's supply and demands obey

   $$\frac{dy_i}{dp_j} = \frac{dy_j}{dp_i} \,?$$

   For what commodities i, if any, is it true that this firm's supply and demands obey

   $$\frac{dy_i}{dp_i} \geq 0 \,?$$

2. **[8 points]**

   (a) If the price vector faced by a price-taking consumer changes from $\mathbf{p}$ to $\lambda\mathbf{p}$, what change in income would leave utility unchanged? Why?

   (b) Suppose a price-taking consumer has a utility function

   $$u(\mathbf{x}) = 2\ln x_1 + \ln x_2$$

   over two goods x_1 and x_2. If the price of only the first good rises, what change in income would leave utility unchanged?

Section 3.
Answer two of the following three questions.

1. **[9 points]**

> The postulate of *methodological individualism* underlies all public choice analysis. In trying to explain governmental actions, we begin by analyzing the behavior of the individuals who make up the government. In a democracy these are the voters, their elected representatives, and appointed bureaucrats. The postulate of methodological individualism has a normative analogue. The actions of government ought to correspond, in some fundamental way, to the preferences of the individuals who these actions effect, the citizens of the state. The postulate of *normative individualism* underlies much of normative analysis in public choice.
> —Dennis Mueller

Explain the term methodological individualism. Discuss its role in neoclassical microeconomics. Discuss and evaluate the alternative schools of thought concerning methodology.

2. **[9 points]**

Kim, Khloe and Kourtney share an apartment at Quasilinear Gardens. They each derive benefit from apartment cleanliness G, a public good, and leisure x, a private good. Their utility functions and endowments are

$$\begin{array}{llll} \text{Kim,} & U_i = x_i + \frac{1}{6}\ln(G), & \omega_i = 1 \\ \text{Khloe,} & U_h = x_h + \frac{1}{3}\ln(G), & \omega_h = 1 \\ \text{Kourtney,} & U_o = x_o + \frac{1}{2}\ln(G), & \omega_o = 1. \end{array}$$

Each citizen may make a contribution g toward the provision of cleanliness, but such contributions reduce private consumption according to the budget constraint $\omega = g + x$.

Production of cleanliness and leisure takes place according to the transformation function

$$T(G, x_i, x_h, x_o) = 0 = G + x_i + x_h + x_o - \omega_i - \omega_h - \omega_o\,.$$

(a) Find the Nash equilibrium; express your answer as (G, x_i, x_h, x_o).

(b) Find the Lindahl equilibrium.

(c) Find the Bowen equilibrium.

3. **[9 points]**
Comcast and Direct TV produce TV service x by combining capital and labor. Their cost functions are

$$c(v_i, w_i, x_i) = (v_i + w_i)\,x_i \quad \text{for } i = c, d\,,$$

where v is the rental rate for capital, w the wage rate. Suppose that inverse demand is given by $p(x) = 8 - x$, where $x = x_c + x_d$ is the quantity of TV service.

(a) Suppose these two firms share the market, and that they behave as a Cournot duopoly. Both face the same costs in competitive factor markets. The rental rate for capital is $v_i = 1$; the wage rate is $w_i = 1$. What is the equilibrium? Illustrate your answer with best response curves.

(b) Now suppose that Comcast's workers unionize and succeed in quadrupling their wage, $w_c = 4$. Find the new equilibrium.

(c) Construct a welfare analysis of these two equilibriums, (a) and (b), to determine which is the more efficient market structure. Calculate the monetary value of this difference.