There are 100 points possible on this exam, 50 points each for Prof. Lozada's questions and Prof. Govindan's questions.

There are three sections on this exam:

- In the first section contains all of the required questions. There are five of them. The first two (from Prof. Lozada) are worth 19 points each; the last two (from Prof. Govindan) are worth 12 points each.
- In the second section there are two questions from Prof. Govindan; you should work one of them. Each is worth 14 points.
- In the third section there are two questions from Prof. Lozada; you should work one of them. Each is worth 12 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we may grade looking at black-and-white photocopies of your exam.

Answers with illegible or difficult-to-read-handwriting may lower your grade because we may not be able to read and understand your answers, especially considering that we are looking at copies. So it is in your best interest to make your answers LEGIBLE.

Please put the number of the problem you are working on at the top of every page of your answers, so we do not accidentally ignore part of your answer.

In this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

If you think there is some ambiguity in a question, spell out exactly how you interpret that question.

A correct answer will not receive full credit without supporting explanations. Show all your work to receive full credit.

Good luck.

Section 1. Answer all of the following five questions.

1. **[19 points]** A competitive firm buys inputs $x_1 > 0$ and $x_2 > 0$ at exogenously given (that is, "fixed") prices w_1 and w_2, and combines them in order to produce good y according to the production function

$$y = x_1^{\alpha} x_2^{\beta}$$

where $\alpha > 0$ and $\beta > 0$. It then sells y at a fixed price p.

   (a) (10 points) What is the effect of a change in w_1 on y? (Hint: You may want to find out what happens to x_1 and to x_2 but you do not have to.) Do not expand any determinants for this part of the question; you may leave the determinants unevaluated.

   (b) (9 points) For what values of α and β is your answer to part (a) valid? You should not leave determinants unevaluated in this part of the question.

2. **[19 points]** Consider a Robinson Crusoe economy (that is, a one-person economy) in which Robinson Crusoe's utility function is

$$u(a, b, R) = 2\sqrt{a} + 2\sqrt{b} + 2\sqrt{R}$$

where a is his consumption of apples, b is his consumption of bananas, and R is his consumption of "leisure" or "*r*est." Out of 24 hours in a day, Robinson spends some in rest, some in labor to produce apples, ℓ_a, and the remainder in labor to produce bananas, ℓ_b. In his role as a consumer, Robinson takes all prices as given.

Let the price of apples be the numéraire, let the price of bananas be p_b, and let the price of labor be the wage rate, w.

In Robinson's role as producer of apples, suppose he earns π_a in profit. In Robinson's role as producer of bananas, suppose he earns π_b in profit.

   (a) (4 points) Show that Robinson's demand for apples is

$$a^D = \frac{w p_b \left[\pi_a + \pi_b + 24w\right]}{p_b + w p_b + w},$$

   Robinson's demand for bananas is

$$b^D = \frac{w \left[\pi_a + \pi_b + 24w\right]}{p_b \left(p_b + w p_b + w\right)},$$

and Robinson's supply of labor is

$$\ell_a + \ell_b = 24 - \frac{p_b\,[\pi_a + \pi_b + 24w]}{(w\,(p_b + wp_b + w)}.$$

(b) (1 point) Suppose that in Robinson's role as producer of apples, Robinson is a price taker (is "perfectly competitive"). Suppose the production function for apples is $a = \ell_a$. Argue that:

   i. The supply function is not "everywhere well-defined."
   ii. The equilibrium wage is one.
   iii. The equilibrium profit from producing apples is zero.

(c) (1 point) Suppose that in Robinson's role as producer of bananas, Robinson is a price taker (is "perfectly competitive"). Suppose the production function for bananas is $b = \ell_b$. Argue that:

   i. The equilibrium price of bananas is one.
   ii. The equilibrium profit from producing bananas is zero.

(d) (2 points) Show that in the general equilibrium of this economy, $a^* = 8$, $b^* = 8$, $\ell_a^* + \ell_b^* = 16$, and $R^* = 8$.

(e) Now suppose that in Robinson's role as producer of bananas, Robinson is *not* a price taker, but rather acts as a monopolist. This means that he knows what the demand curve for bananas is (it is given in part (a) above). He also completely understands how the demand for bananas depends on the profit from producing bananas. Nothing changes about Robinson's role as producer of apples; it is still as described in part (b) above. Find the equilibrium:

   i. (1 point) wage rate and the profit from producing apples;
   ii. (4 points) price of bananas (hint: it is the solution to $0 = 2p_b^2 - 4p_b - 1$, and you get full credit for showing that; you do not have to show the solution to that equation, which is $p_b = 1 + (\sqrt{6}/2) \approx 2.22$, but you can use this result in the other parts of this problem);
   iii. (2 points) number of bananas produced (hint: it is

$$\frac{6\sqrt{6} + 12}{(1 + \frac{\sqrt{6}}{2})\,(3 + \sqrt{6})},$$

and you get full credit for showing that or an equivalent numerical expression; you do not have to show that that simplifies to $12 - 4\sqrt{6} \approx 2.20$, but you can use this result in the other parts of this problem);

iv. (2 points) number of apples produced (hint: it is

$$\frac{(1 + \frac{\sqrt{6}}{2})\,(6\sqrt{6} + 12)}{3 + \sqrt{6}},$$

and you get full credit for showing that or an equivalent numerical expression; you do not have to show that that simplifies to $2\,(\sqrt{6} + 3) \approx 10.9$, but you can use this result in the other parts of this problem);

v. (1 point) amount of rest R (hint: it is $6 + 2\sqrt{6} \approx 10.9$, which you should show).

(f) (1 point) Write a summary of the effect *on the market for apples* and *on the market for labor* when the banana market becomes monopolized.

3. **[12 points]** Consider an investor who must decide how much of his initial wealth w to put into a risky asset. The risky asset can have any of the positive or negative rates of return r_i with probabilities p_i, $i = 1, \ldots, n$. If β is the amount of wealth to be put into the risky asset, final wealth under outcome i will be $(w - \beta) + (1 + r_i)\beta = w + \beta r_i$. The investor's problem is to choose β to maximise the expected utility of wealth. Assume the utility function of investor $u(\cdot)$ to be strictly increasing in wealth and twice continuously differentiable.

   (a) (3 points) Write the investor's problem as a single-variable optimisation problem.

   (b) (4 points) Under what conditions will a risk-averse investor decide to put *no* wealth into the risky asset?

   (c) (5 points) Assume the risky asset has a positive expected return, the optimal investment $\beta^* < w$, and the investor is risk-averse. Find the optimal investment β^* under these assumptions.

4. **[12 points]** Consider the strategic form game depicted below. Each of two countries must simultaneously decide on a course of action.

Country 1 must decide whether to keep its weapons or to destroy them. Country 2 must decide whether to spy on country 1 or not. It would be an international scandal for country 1 if country 2 could prove that country 1 was keeping its weapons. The payoff matrix is as follows.

	Spy	Don't Spy
Keep	$-1, 1$	$1, -1$
Destroy	$0, 2$	$0, 2$

(a) (2 points) Does either player have a strictly dominant strategy?

(b) (2 points) Does either player have a weakly dominant strategy?

(c) (8 points) Find a Nash equilibrium in which neither player employs a weakly dominant strategy. (Hint: Consider both pure and mixed strategy equilibrium to solve this question.)

5. **[12 points]** Reconsider the two countries from the previous question, but now suppose that Country 1 can be one of two types, aggressive or non-aggressive. Country 1 knows its own type. Country 2 does not know Country 1's type, but believes that Country 1 is aggressive with probability $\epsilon > 0$. The aggressive type places great importance on keeping its weapons. If it does so and Country 2 spies on the aggressive type this leads to war, which the aggressive type wins and justifies because of the spying, but which is very costly for Country 2. When Country 1 is non-aggressive, the payoffs are as before (i.e., as in the previous question). The payoff matrices associated with each of the two possible types of country 1 are given below.

Country 1 is 'aggressive'
Probability ε

	Spy	Don't Spy
Keep	$10, -9$	$5, -1$
Destroy	$0, 2$	$0, 2$

Country 1 is 'non-aggressive'
Probability $1 - \varepsilon$

	Spy	Don't Spy
Keep	$-1, 1$	$1, -1$
Destroy	$0, 2$	$0, 2$

(a) (2 points) What action must the aggressive type of Country 1 take in any Bayesian-Nash equilibrium?

(b) (10 points) Find the pure-strategy Bayes-Nash equilibrium if $\epsilon \geq 1/5$.

Section 2. Answer one of the following two questions.

1. **[14 points]** Consider the following signaling game. An item is of high quality with probability $1/3$ and of low-quality with probability $2/3$. The seller of the item is privately informed of its quality. The seller moves first, choosing whether to advertise ("adv") or not ("none"). The buyer observes whether or not the seller advertises and then chooses whether or not to buy. The price is fixed at \$3, whether the seller advertises or not. The cost of advertising for the seller is \$1 for the high quality item and is \$4 for the low quality item. The buyer values the item at \$4 if it is high quality and at \$2 if it is low quality.

   The seller's payoff is sales revenue minus advertising costs (if any). The buyer's payoff is the difference between his value for the item and the price he pays. Assume buyer's payoff to be zero if he does not buy the item.

   (a) (4 points) Sketch the extensive form representation of the game with payoffs for both the players.

   (b) (10 points) Find all the pure strategy Perfect Bayesian Equilibria of the game. Show all your work for all the possible pure strategy profiles whether or not that profile is equilibrium.

2. **[14 points]** There are two "types" of workers: HIGH ability ($\theta = 2$) and LOW ability ($\theta = 1$), where θ measures ability. Employers don't know the type of any one worker but have commonly known prior beliefs: $Pr(\theta = 1) = 1/3$ and $\Pr(\theta = 2) = 2/3$. Productivity of any worker is 2θ and cost of education e is $C(e) = e/\theta$. First, the worker chooses the level of education, e. The employer, upon observing e, chooses the wage. Assume the wage equals to expected productivity.

   Find all the pure strategy separating and pooling Perfect Bayesian Equilibria. Assume that when employers observe education $e \neq e_H$ in a separating equilibrium, they believe worker is LOW type for sure, where e_H is the education choice of the HIGH ability worker. Assume that when employers observe education $e \neq e^*$ in a pooling equilibrium, they believe that worker is LOW type for sure, where e^* is the pooling strategy for the HIGH and LOW ability workers.

Section 3. Answer one of the following two questions.

1. **[12 points]** Suppose a consumer gets utility from consumption of apples "a" and bananas "b" according to a quasiconcave utility function $u(a, b)$. Suppose the consumer's initial consumption bundle is (a_0, b_0), and let U_0 be $u(a_0, b_0)$ where u is increasing in a and b. The purpose of this question is to investigate one way to define the "value" to this consumer of giving this consumer more apples, moving his consumption bundle to (a_1, b_0), where $a_1 > a_0$.

   (a) (2 points) Let compensating variation "CV" for this environment which lacks prices and incomes be implicitly defined by

   $$u(a_0, b_0) = u(a_1, b_0 - CV).$$

   (In microeconomic theory textbooks, CV is only defined in environments with prices and incomes.) How could CV, defined in this way, be interpreted as a measure of the value of moving from a_0 to a_1?

   (b) (2 points) Sketch a graph with a on the horizontal axis and b on the vertical axis, illustrating CV. Hint: begin by drawing the indifference curve which $u(a_0, b_0)$ lies on.

   (c) (3 points) By calculating the appropriate (total) differential, show that

   $$\frac{\partial\, CV}{\partial\, a_1} = \frac{\partial u(a_1, b_0 - CV)/\partial a_1}{\partial u(a_1, b_0 - CV)/\partial(b_0 - CV)}$$

   which is an abbreviated notation for

   $$\frac{\partial\, CV}{\partial\, a_1} = \frac{\left.\dfrac{\partial u(a,b)}{\partial a}\right|_{(a_1, b_0 - CV)}}{\left.\dfrac{\partial u(a,b)}{\partial b}\right|_{(a_1, b_0 - CV)}}. \tag{1}$$

   (d) (3 points) As a_1 increases, does the quasiconcavity of u imply that the right-hand side of (1) increases, decreases, or remains constant? Why? (If you make an assertion about how the quasiconcavity of u affects the indifference curves, you do not have to prove that assertion.) Hint: Rather than calculate any derivatives, think about what happens to the Marginal Rate of Substitution of a for b. It is completely acceptable for you to simply assert, rather than rigorously prove, the connection between the Marginal Rate of Substitution and this problem, because that proof is an undergraduate-level exercise.

(e) (2 points) Sketch a rough graph of CV versus a_1. Make sure the first derivative of your sketch of $CV(a_1)$ is consistent with (1) and the second derivative of your sketch of $CV(a_1)$ is consistent with your answer to part (d).

2. **[12 points]** Baby Alice has two grandfathers, Grandpa Smith and Grandpa Jones. Suppose there is one good, called "x", and suppose the consumption of that good by Baby Alice, Grandpa Smith, and Grandpa Jones is x_A, x_S, and x_J, respectively.

Grandpa Smith has an endowment of 3 units of good x, of which he consumes x_S and gives x_{SA} to Baby Alice:

$$x_S + x_{SA} = 3 .$$

Grandpa Jones has an endowment of 3 units of good x, of which he consumes x_J and gives x_{JA} to Baby Alice:

$$x_J + x_{JA} = 3 .$$

Baby Alice has no endowment of good x; all of her consumption consists of gifts from her grandfathers:

$$x_A = x_{SA} + x_{JA} .$$

Baby Alice's utility function is

$$u_A(x_A) = \ln x_A .$$

Grandpa Smith's utility function is

$$u_S(x_S, u_A) = \ln x_S + u_A .$$

Grandpa Jones's utility function is

$$u_J(x_J, u_A) = \ln x_J + u_A .$$

(So the grandfathers care about their own consumption and the utility of Baby Alice.)

(a) (5 points) Find the values of x_{SA}, x_{JA}, x_A, x_S, x_J, u_S, u_J, and u_A. Assume that each grandfather takes the other grandfather's actions as fixed. (So this is technically an equilibrium in the sense of Nash.)

(b) (5 points) Denote the values found in part (a) as x^*_{SA}, x^*_{JA}, x^*_A, x^*_S, x^*_J, u^*_S, u^*_J, and u^*_A. Now suppose that instead of giving x^*_{SA} and x^*_{JA}, each grandfather increases his gift by $\epsilon > 0$. Show that for sufficiently small ϵ, this change increases *everyone's* utility compared with the situation of part (a).

(c) (1 point) Is the Nash Equilibrium efficient? Why or why not?

(d) (1 point) Briefly speculate about the welfare implications of this example if, instead of Baby Alice having two grandfathers, she had four great-grandfathers.