Summary 2021 Microeconomics Qualifying Exam

There are 100 points possible on this exam, 50 points each for Prof. Lozada's questions and Prof. Govindan's questions.

There are three sections on this exam:

- In the first section contains all of the required questions. There are five of them. The first two (from Prof. Lozada) are worth 20 points each; the last three (from Prof. Govindan) are worth 12 points each.
- In the second section there are two questions; you should work one of them. Each is worth 14 points.
- In the third section there are two questions; you should work one of them. Each is worth 10 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we may grade looking at black-and-white photocopies of your exam.

Answers with illegible or difficult-to-read-handwriting may lower your grade because we may not be able to read and understand your answers, especially considering that we are looking at photocopies. So it is in your best interest to make your answers readable.

Please put the number of the problem you are working on at the top of every page of your answers, so we do not accidentally ignore part of your answer.

In this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

Good luck.

Section 1.
Answer all of the following five questions.

1. **[20 points]** Suppose a competitive profit-maximizing firm uses gasoline and engines to produce output "y." Suppose a new type of engine is developed which can produce the same output using less gasoline. The best way to model this "technological progress" would be to assume that the production function changed from

$$y = \hat{f}(\text{gasoline, old engines})$$

to

$$y = \hat{g}(\text{gasoline, old engines, new engines})$$

for old and new production functions $\hat{f}$ and $\hat{g}$. However, an easier way to model this technological change would be to use one production function "f,"

$$y = f(\alpha \cdot \text{gasoline, engines}),$$

where α is a parameter indicating the state of technology, and where the distinction between old and new engines is ignored (they are both "engines"). Use this production function "f" when answering this question.

   (a) Would technological progress be indicated by α rising or falling? Why?

   (b) Assuming all prices remain unchanged, will technological progress (as measured by α) cause the demand for gasoline to rise or to fall?

2. **[20 points]** Consider a two-person two-commodity competitive general equilibrium in which the two people are denoted by 1 and 2, the two goods by x and y, and person i consumes x_i of good x and y_i of good y. Suppose the utility functions of the two individuals are

$$U_1 = x_1^{\alpha} y_1^{\beta} \quad \text{and}$$
$$U_2 = x_2^{\gamma} y_2^{\delta}.$$

Suppose α, β, γ, and δ are all positive. Suppose both people have 1 unit of good x and 1 unit of good y as their endowment (in other words, $\omega_1 = \omega_2 = (1, 1)$).

What effect will an increase in α have on the amount of good y which person 2 consumes?

Hint: The following answer is *wrong:* "an increase in α has no effect on the amount of good y which person 2 consumes."

3. **[12 points]** John is an expected utility maximizer with vNM utility function $u(x) = \sqrt{x}$.

   (a) (3 points) John currently holds a portfolio of stocks. If the economy tanks, these stocks will be worthless and his wealth will be zero. If the economy does not tank, his wealth will be \$1,000,000. The probability that the economy will tank is 1/10. Calculate the certainty equivalent of John's current holdings.

   (b) (3 points) Suppose that John can buy insurance such that he will pay \$$X$ to the insurer if the economy does not tank and he will receive a payment of \$1,000,000 − \$$X$ from the insurer if the economy tanks. What is the largest amount \$$X$ that John would be willing to pay for this insurance?

   (c) (3 points) What is the largest amount that John would be willing to pay for the insurance if his vNM utility function were linear in his wealth?

   (d) (3 points) Suppose John's initial wealth is W. Show either analytically or graphically that

   $$0.5\, u(W-1) + 0.5\, u(W+1) \geq 0.5\, u(W-2) + 0.5\, u(W+2)$$

   where u is John's vNM utility function.

4. **[12 points]** Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = cq_i$, but demand is uncertain: it is high ($a = a_H$) with probability θ and low ($a = a_L$) with probability $1 - \theta$. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. All of this is common knowledge. The two firms simultaneously choose quantities. What are the strategy spaces for the two firms? Make assumptions concerning a_H, a_L, θ, and c such that all equilibrium quantities are positive. What is the Bayesian Nash Equilibrium of this game?

5. **[12 points]** Two investors have deposited D with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of $2r$ can be recovered, where $D > r > D/2$. If the bank allows the investment to reach maturity, however, the project will pay out a total of $2R$,where $R > D$.

   There are two dates at which the investors can make withdrawals from the bank: date 1 is before the bank's investment matures; date 2 is after. Assume that there is no time discounting. If both investors make withdrawals at date 1 then each receives r and the game ends. If only one investor makes a withdrawal at date 1 then that investor receives D, the other receives $2r - D$, and the game ends. Finally, if neither investor makes a withdrawal at date 1 then the project matures and the investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2 then each receives R and the game ends. If only one investor makes a withdrawal at date 2 then that investor receives $2R - D$, the other receives D, and the game ends. Finally, if neither investor makes a withdrawal at date 2 then the bank returns R to each investor and the game ends.

   (a) (6 points) Give the extensive and normal form representations of the bank run game discussed above.

   (b) (6 points) What are the pure-strategy subgame perfect Nash equilibria?

Section 2.
Answer one of the following two questions.

1. **[14 points]** Consider the following signaling game. An item is of high quality with probability $1/3$ and of low-quality with probability $2/3$. The seller of the item is privately informed of its quality. The seller moves first, choosing whether to advertise ("adv") or not ("none"). The buyer observes whether or not the seller advertises and then chooses whether or not to buy. The price is fixed at \$3, whether the seller advertises or not. The cost of advertising for the seller is \$1 for the high quality item and is \$4 for the low quality item. The buyer values the item at \$4 if it is high quality and at \$2 if it is low quality.

   The seller's payoff is sales revenue minus advertising costs (if any). The buyer's payoff is the difference between his value for the item and the price he pays. Assume buyer's payoff to be zero if he does not buy the item.

   (a) (4 points) Sketch the extensive form representation of the game with payoffs for both the players.

   (b) (10 points) Find all the pure strategy Perfect Bayesian Equilibria (PBE) of the game. Show all your work for all the possible pure strategy profiles whether or not that profile is equilibrium.

2. **[14 points]** There are two "types" of workers: HIGH ability ($\theta = 2$) and LOW ability ($\theta = 1$), where θ measures ability. Employers don't know the type of any one worker but have commonly known prior beliefs: $Pr(\theta{=}1) = 1/3$ and $\Pr(\theta{=}2) = 2/3$. The productivity of a worker is 2θ and the cost of education e is $C(e) = e/\theta$. First, the worker chooses a level of education, e. The employer, upon observing e, chooses a wage. Assume the wage equals to expected productivity.

   Find all the pure strategy separating and pooling Perfect Bayesian Equilibria. Assume that when employers observe education $e \neq e_H$ in a separating equilibrium, they believe that worker is the LOW type for sure, where e_H is the education choice of the HIGH ability worker. Assume that when employers observe education $e \neq e^*$ in a pooling equilibrium, they believe that worker is the LOW type for sure, where e^* is the pooling strategy for the HIGH and LOW ability workers.

Section 3.
Answer one of the following two questions.

1. **[10 points]** Let p^0 be an "initial" price vector, p' be a "final" price vector, m^0 be "initial" income, and m' be "final" income. One definition of equivalent variation, EV, and compensating variation, CV, is

$$EV = e(p^0, v(p', m')) - m^0$$
$$CV = m' - e(p', v(p^0, m^0)$$

where $e(\cdot)$ denotes the expenditure function of a consumer and $v(\cdot)$ denotes the indirect utility function of that consumer. Let $u^0 = v(p^0, m^0)$ and $u' = v(p', m')$. Under the assumption that $m^0 = m'$, Varian writes on his page 167 that:

$$EV = e(p^0, u') - e(p', u')$$
$$CV = e(p^0, u^0) - e(p', u^0).$$

Finally, using the fact that the Hicksian demand function is the derivative of the expenditure function, so that $h(p, u) \equiv \partial e/\partial p$, we can write these expressions as

$$\begin{aligned} EV &= e(p^0, u') - e(p', u') = \int_{p'}^{p^0} h(p, u')\, dp \\ CV &= e(p^0, u^0) - e(p', u^0) = \int_{p'}^{p^0} h(p, u^0)\, dp. \end{aligned} \tag{10.2}$$

It follows from these expressions that the compensating variation is the integral of the *Hicksian* demand curve associated with the initial level of

Show that if $m^0 \neq m'$, Varian's (10.2) have to be modified to

$$EV = \int_{p'}^{p^0} h(p, u')\, dp + \text{something more}$$
$$CV = \int_{p'}^{p^0} h(p, u^0)\, dp + \text{something more}\,.$$

by finding what the "something more" parts are.

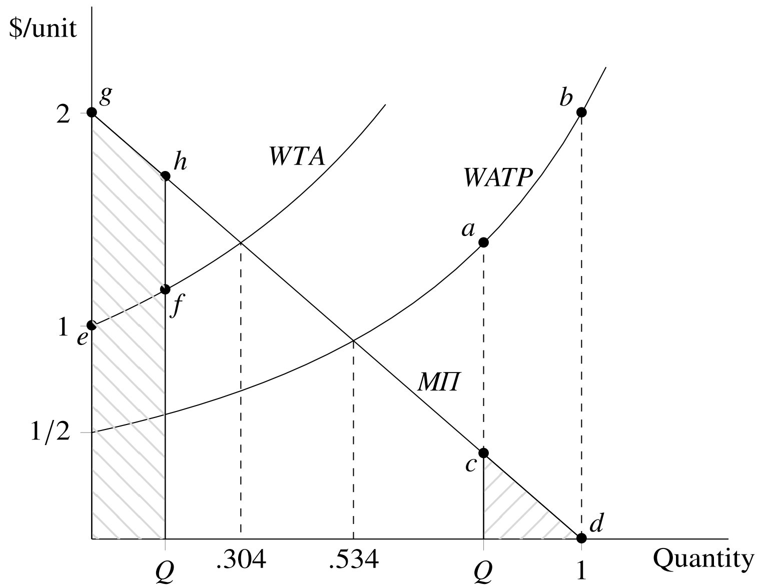

Figure 1. Willingness (and ability) to Pay, "*WATP*," is $m\overline{Q}/[2\cdot(Q-\overline{Q})^2]$ from (i) of the 2019 exam, and $2/(Q-2)^2$ substituting in the parameters here. Willingness to Accept, "*WTA*," is $m\overline{Q}/(\overline{Q}-Q)^2)$ from (l) (that's the letter 'l' not the number '1') of the 2019 exam, and $4/(2-Q)^2$ substituting in the parameters here. Marginal Profit, "$M\Pi$," is assumed here to be $2-2Q$.

2. **[10 points]**

   [Completely optional introduction: This is the end of a demonstration that George Stigler's "Coase Theorem" is false in the general case of goods having arbitrary income effects. (This would please Nobel Laureate Ronald Coase but it profoundly challenges followers of Stigler.)]

   You have been given the question and answer to the 2019 Qualifying Exam's Section 3 Question 1. Adopt all the notation and situations described in that problem. In addition, in that problem, set $\overline{Q} = 2$, $m = 2$, and suppose the marginal profit of the firm is given by $M\Pi = 2-2Q$. The figure on the last, "optional" page of the answer to the 2019 question then becomes Figure 1.

   (a) Suppose, as in part (c) of the 2019 question, firms have the right to emit pollution at will. The maximum amount of money the consumer is willing and able to pay for the output level "Q" lying directly below points c and a is the area under the WATP curve, that is, under ab. However, in bargaining with the firm, the consumer might not have to pay this maximum amount of money. Explain briefly why the *minimum* amount of money the consumer would have to pay for the output to be reduced to that level of Q is the area under the "cd" segment of the $M\Pi$ curve.

(b) Show that the area under the cd segment of the $M\Pi$ curve is $1 - 2Q + Q^2$.

(c) From (c) of the 2019 exam, $v_0 = m\overline{Q}/(2p_a)$, which under our assumptions is equal to $2/p_a$. From (d) of the 2019 exam, and with our assumptions,

$$v' = \frac{m - T}{p_a}\left(\overline{Q} - Q\right) = \frac{2 - T}{p_a}(2 - Q)\,.$$

If T corresponds not to WATP but to the minimum consumer payment, what value of Q makes v_0 equal to v'? It is sufficient to find an equation that defines Q implicitly; you do not have to find Q explicitly.

(d) Suppose, as in part (j) of the 2019 question, consumers have the right to clean air and firms cannot pollute the air without obtaining permission from the consumer. The minimum amount of money the consumer is willing to accept for the output level "Q" lying directly below points h and f is the area under the WTA curve, that is, under ef. However, in bargaining with the firm, the consumer might not have to accept this minimum amount of money. The *maximum* amount of money firm would pay to the consumer for the output to be increased to that level of Q is the area under the "gh" segment of the $M\Pi$ curve. Show that that area under the gh segment of the $M\Pi$ curve is $2Q - Q^2$.

(e) From (j) of the 2019 exam, $v_0 = m\overline{Q}/p_a$, which under our assumptions is equal to $4/p_a$. From (k) of the 2019 exam, and with our assumptions,

$$v' = \frac{m + \widehat{T}}{p_a}\left(\overline{Q} - Q\right) = \frac{2 + \widehat{T}}{p_a}(2 - Q)\,.$$

If $\widehat{T}$ corresponds not to WTA but to the maximum firm payment, what value of Q makes v_0 equal to v'? It is sufficient to find an equation that defines Q implicitly; you do not have to find Q explicitly.