There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are weighted differently from Prof. Kiefer's questions: Prof. Lozada's questions are worth 14 points, 14 points, and 8 points, while Prof. Kiefer's questions are worth 18 points, 9 points, and 9 points.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 14 points; the second is worth 14 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 8 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we grade looking at black-and-white photocopies of your exam.

It would be helpful for you to put the number of the problem you are working on at the top of every page of your answers.

In this document,

$$v, \quad w, \quad \omega, \quad \boldsymbol{\omega}$$

denote, in the order in which they appear, the Roman lower-case "v" and "w" and the Greek lower-case "omega" and "boldface omega." Also, in this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

Good luck.

Section 1.
Answer all of the following three questions.

1. **[14 points]** Suppose a competitive, profit-maximizing firm transforms two inputs (x_1 and x_2) into one output (y) according to a well-behaved, concave, fully differentiable production function $f(x_1, x_2)$. Let the price of the inputs be p_1 and p_2 and let the price of the output be w.

   Suppose that the government introduces a tax (t) on each unit of x_1 bought. In other words, assume this tax is a "specific tax," such as \$0.70/unit, not an "ad valorem tax," which would be expressed as a percentage such as 7%.

   Feel free to use abbreviations to simplify the answers you derive below.

   (a) Assuming no prices change (that is, p_1, p_2, and w do not change), how will this tax change:

      i. the firm's demand for the taxed input x_1;
      ii. the firm's demand for the untaxed commodity x_2; and
      iii. the supply of the output y?

   (b) How will a change in the price of x_1 affect the demand for x_1?
   (c) How will a change in the price of x_1 affect the demand for x_2?
   (d) How will a change in the price of x_1 affect the supply of y?
   (e) How will a change in the price of x_2 affect the demand for x_1?
   (f) How will a change in the price of x_2 affect the demand for x_2?
   (g) How will a change in the price of x_2 affect the supply of y?
   (h) How will a change in the price of y affect the demand for x_1?
   (i) How will a change in the price of y affect the demand for x_2?
   (j) How will a change in the price of y affect the supply of y?
   (k) Now suppose that all competitive firms producing y use x_1 and x_2 and are subject to this tax on x_1. Using Cramer's Rule, derive an expression for the effect of this tax on the equilibrium price of x_1 when all prices are allowed to change.

      Your answer will involve a 3×3 determinant; you should leave it unevaluated to save time. Also to save time, if your answer involves quantities which you derived in parts (a)–(j), you can just write, for example, "(h)" instead of writing in the answer which you found in part (h). As a final time-saving measure, just assume the number of firms producing y is equal to one even though that is a strange assumption because the firm(s) is (are) competitive.

(A similar question appeared on a previous exam in a past year, and the answer I gave for it only involved a 2×2 determinant, but that answer should have taken one more market into account, and if it had done so, it would have involved a 3×3 determinant as well.)

2. **[14 points]** Suppose an economy consists of two agents, "a" and "b," and two goods, "1" and "2," and the agents have the following utility functions and endowments:

$$\begin{aligned} u_a &= \ln(x_{1a}) + x_{2a} \qquad & \omega_a &= (1,1), \\ u_b &= \ln(x_{1b}) + x_{2b} \qquad & \omega_b &= (9,9). \end{aligned}$$

(a) Find the competitive equilibrium prices (or price ratio) and allocation $(x_{1a}^*, x_{2a}^*, x_{1b}^*, x_{2b}^*)$ for this economy. You may work either with the price of Good 1, "p_1," and the price of Good 2, "p_2," or with their ratio (for example, $\rho = p_2/p_1$ or $\gamma = p_1/p_2$), or you may choose a numéraire.

(b) Suppose that while the behavior of Person "b" is identical to that in part (a), Person "a" now behaves in the following noncompetitive way regarding Good 2:

- Person "a" knows that $x_{2a} + x_{2b} = 10$;
- Person "a" knows the demand curve for Good 2 by Person "b" (which you worked out in part (a)); and
- Person "a" can choose p_2 (or, equivalently, Person "a" can choose ρ or γ).

i. Find the demand by Person "a" for Good 2 as a function of price(s). You should be able to do this without solving an optimization problem by taking into account the first and second "bullet points" above.

ii. Find the resulting non-competitive equilibrium prices (or price ratio) and allocation $(\hat{x}_{1a}, \hat{x}_{2a}, \hat{x}_{1b}, \hat{x}_{2b})$ for this economy. Hint: in working this out, at one point I got to

$$0 = \frac{p_2^2}{9p_1} - 10p_1 + p_2 .$$

If you get to the same point, you should be able to use algebra and the quadratic formula to simplify this to

$$0 = \rho^2 + 9\rho - 90 = (\rho + 15)(\rho - 6)$$

using $\rho = p_2/p_1$ as mentioned in part (a).

(c) It turns out that the equilibrium allocation for part (a) occurs at the point labeled "C" (for "competitive") in the Edgeworth Box illustrated in Figure 1, and that the equilibrium allocation for part (b) occurs at the point labeled "NC" (for "non-competitive") in Figure 1. Figure 1 is not drawn to scale, and it omits portions of the Edgeworth Box in order to better show its lower-lefthand corner. The numbers next to the figure's indifference curves (the numbers 1, 1.81, 1.89, 11.2, 11.3, and 11.4) represent the approximate corresponding value of u_a or u_b.

The straight line through NC represents the non-competitive equilibrium price vector of part (b). At NC, is the indifference curve of Person "b" tangent to this price vector? Why? At NC, is the indifference curve of Person "a" tangent to this price vector? Why?

(d) Suppose that each year, the economy starts at (ω_a, ω_b) and, because Person "a" has market power, each year the economy ends up at point NC. Furthermore, suppose the US Department of Justice has filed a lawsuit against Person "a" in which a court is asked to prohibit Person "a" from engaging in non-competitive behavior. If the Department of Justice is successful, in future years the economy will be at C. If the Department of Justice is unsuccessful, in future years the economy will continue ending up at NC.

i. If you were an economic consultant for the Department of Justice, what argument or arguments might you make to the court?

ii. If you were an economic consultant for Person "a," what argument or arguments might you make to the court?

3. **[18 points]**

Story One: Kim and Kanye consume two private goods, beer x_1, and pizza x_2. The utility functions and endowments are given as follows:

Kim	$U_k = \ln(x_{k1}) + \ln(x_{k2})$	$\omega_k = (0.2, 0.8)$,
Kanye	$U_w = \ln(x_{w1}) + \ln(x_{w2})$	$\omega_w = (0.8, 0.2)$.

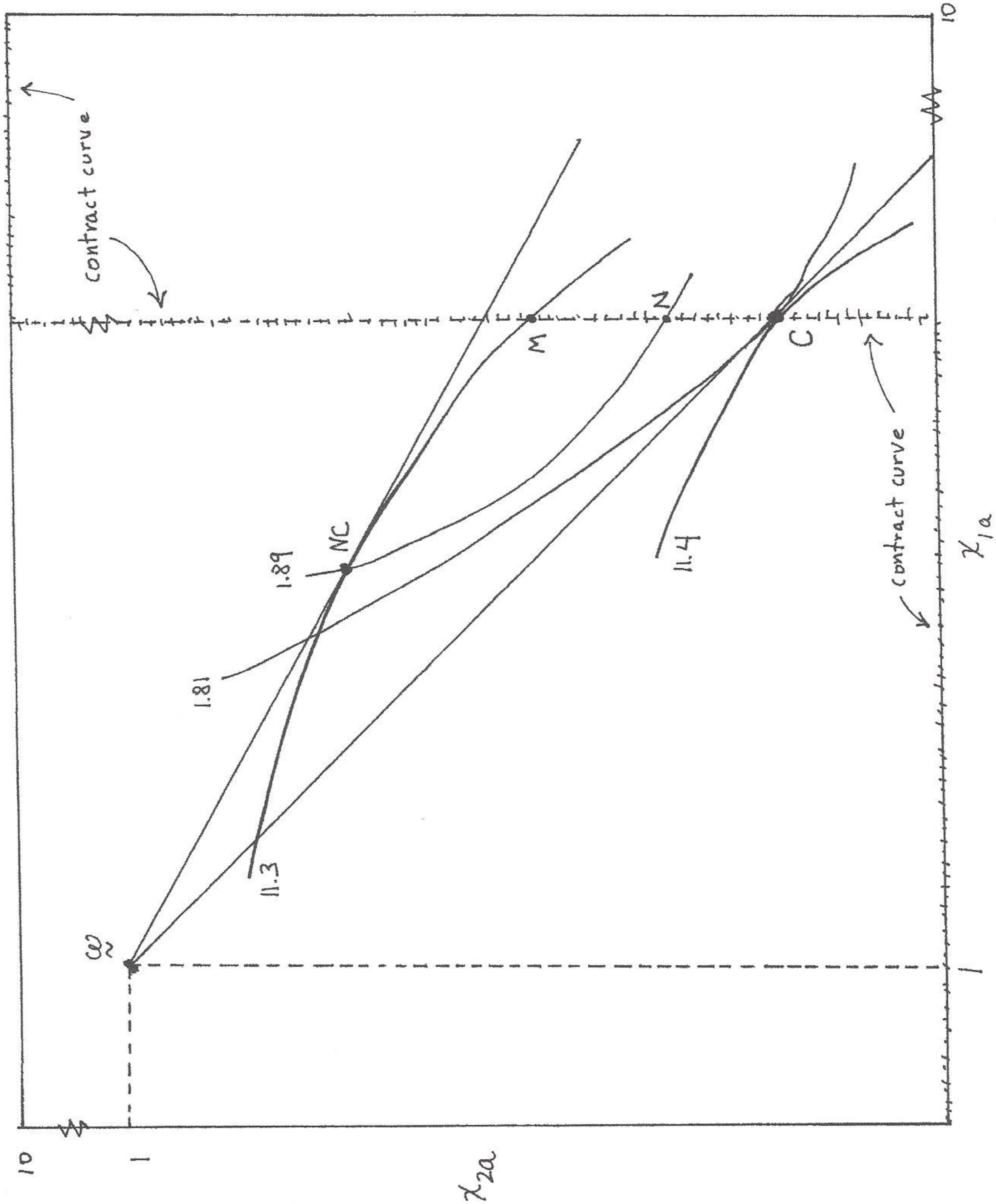
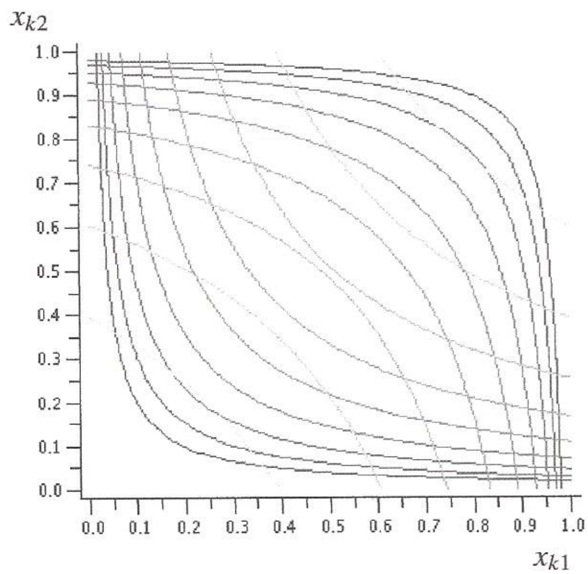

contract curve
contract curve
M
N
C
NC
1.89
1.81
11.3
11.4
ω
10
1
10
1
x2a
x1a

Figure 1

A feasible general equilibrium is described by $0 = \omega_{k1} + \omega_{w1} - x_{k1} - x_{w1}$ and $0 = \omega_{k2} + \omega_{w2} - x_{k2} - x_{w2}$. Kim and Kanye agree on the Benthamite social welfare function,

$$W = U_k + U_w .$$

The diagram below plots the indifference curves of both with respect to Kim's consumption bundle.



(a) Identify the endowment point and the core. Find the Walrasian equilibrium from the given endowment. What is the equilibrium price vector? Illustrate your answer.

(b) Find the Pareto set (contract curve) and the social optimum. Illustrate your answer.

Story Two: Kim and Kanye consume a private good, coffee x_i, and a public good, poetry G. The utility functions and endowments (of coffee) are given as follows:

Kim	$U_k = \ln(x_k) + \ln(G)$	$\omega_k = 1,$
Kanye	$U_w = \ln(x_w) + \ln(G)$	$\omega_w = 1.$

Both may make a contribution g_i toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i .$$

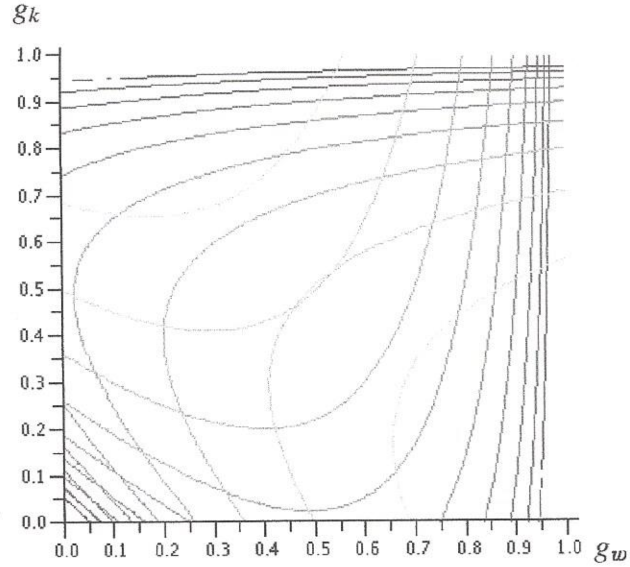
The coffee can be transformed into poetry according to the transformation function

$$0 = x_k + x_w + g_k + g_w - \omega_k - \omega_w \,.$$

Again, Kim and Kanye agree on the Benthamite social welfare function,

$$W + U_k + U_w \,.$$

The diagram below plots the indifference curves of both in contribution space.



(c) Find the Nash and the Lindahl equilibriums. Would both favor a move from the Nash to the Lindahl?

(d) How are the two stories similar? How do they differ? Discuss how the First Theorem of Welfare Economics applies to each.

(e) Find the Pareto set and the social optimum. Illustrate your answer.

(f) Compare the Walrasian and Lindahl equilbriums. Are they guaranteed to be socially optimal? Discuss the wider implications of these stories.

Section 2.
Answer one of the following two questions.

1. **[8 points]**

   Suppose the expenditure function of a consumer is $e(\mathbf{p}, u) = (p_1^r + p_2^r)^{1/r} u$ where p_1 and p_2 are prices, $\mathbf{p}$ is the vector (p_1, p_2), and u is utility. Find this consumer's Marshallian demand curves.

2. **[8 points]**

   (a) Suppose an economy consists of two persons, A, and B. They have initial endowments ω_a and ω_b, respectively. They have consumption bundles $\mathbf{x}_a$ and $\mathbf{x}_b$, respectively. They have utility functions $u_a(\mathbf{x}_a)$ and $u_b(\mathbf{x}_b)$, respectively.

   What procedure would you use to calculate the core of this economy? List all the steps. If there are optimization problems involved, give the Lagrangians and state what the unknowns are. You need not calculate nor state any first-order conditions. I just want you to state a step-by-step procedure which someone else who knew multidimensional optimization but knew nothing about the core could follow to calculate the core, if that person knew what the utility functions were.

   (b) Suppose an economy consists of three persons, A, B, and C. They have initial endowments ω_a, ω_b, and ω_c, respectively. They have consumption bundles $\mathbf{x}_a$, $\mathbf{x}_b$, and $\mathbf{x}_c$, respectively. They have utility functions $u_a(\mathbf{x}_a)$, $u_b(\mathbf{x}_b)$, and $u_c(\mathbf{x}_c)$, respectively.

   What procedure would you use to calculate the core of this economy? List all the steps. If there are optimization problems involved, give the Lagrangians and state what the unknowns are. You need not calculate nor state any first-order conditions. I just want you to state a step-by-step procedure which someone else who knew multidimensional optimization but knew nothing about the core could follow to calculate the core, if that person knew what the utility functions were.

Section 3.
Answer two of the following three questions.

1. **[9 points]** Consider a duopoly strategy game with three options: labeled *left, middle* and *right*. The profit payoff matrix is

profit payoffs: (Airbus, Boeing)		Boeing		
		left	*middle*	*right*
Airbus	*left*	12, 12	5, 14	1,−1
	middle	14, 5	8, 8	2, 0
	right	−1, 1	0, 2	3, 3

   (a) Explain why the *middle* dominates the *left* for both players. Are there any Nash equilibriums in a one-shot, simultaneous game? Explain.

   (b) Two games are played; each is simultaneous. Consider the strategic threat:
   - play *left* in the first game,
   - if rival plays *left* in the first, then play *middle* in second,
   - if rival does not play *left* in the first, then play *right* in second

   Under what circumstances is [1^{st} game: (*left, left*), 2^{nd} game: (*middle, middle*)] a Nash equilibrium?

   (c) Is punishment for deviation [1^{st}: (*left, middle*), 2^{nd}: (*right, right*)] subgame perfect?

2. **[9 points]**
W. W. Norton and Co. holds a monopoly in the production and sale of the Varian textbook. Suppose that Norton can produce any amount of books at the constant marginal (and average) cost of \$20. This monopoly sells books in two different markets, separated by some distance. The demand curve in the first market (North America) is given by

$$x_1 = 180 - p_1 ,$$

and in the second market (rest of the world) is given by

$$x_2 = 180 - 3p_2 .$$

(a) If Norton can maintain separation between these markets, how many books should be produced for each market, and what prices should be charged? What profit results?

(b) How would your answer change if shipping costs between the two markets were zero so that Norton is forced to follow a single-price policy? Now what are the monopolist's quantities, price, profit?

(c) How would your answer change if the shipping costs $40 per book?

3. **[9 points]**

A democratic society consists of many citizens, identical except for their employment status. There are only two time periods: the present ($t = 1$) and the future ($t = 2$). Each individual has the following utility function,

$$U^j = E\left(\ln(c_1^j) + \ln(c_2^j)\right) \quad \text{for } j \in \{\text{employed, unemployed}\}$$

where j denotes employment status: $j = e$ for employed, $j = u$ for unemployed, and c_t^j is consumption in the t^{th} period.

The unemployment rate in period 1 is $u_1 = 0.10$. The probability that an employed in period 1 will lose her job for period 2 is $\phi = 0.04$ (the firing rate), while the probability that an unemployed will gain a job is $\nu = 0.36$ (the hiring rate).

An election during the first period sets a tax τ on the employed during the second period to finance the unemployment insurance benefit f. Total tax collections equal benefits paid. In the first period employed consumption is $c_1^e = 1$, and unemployed consumption is $c_1^u = 0$; in the second period $c_2^e = 1 - \tau$ and $c_2^u = f$. On election day voters know their employment status in period 1, but not in period 2.

(a) What tax does the employed majority prefer? What is the implied benefit level? Explain why the adverse selection issues are not relevant in this example.

(b) A social planner has a Benthamite welfare function defined on the allocation in period 2. What tax and benefit level would this planner prefer?

(c) Discuss the relevance of a political-economic equilibrium as an extension economic theory.