There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, as you can see below, Prof. Lozada's questions are equally weighted, while Prof. Kiefer's required question is worth twice as much as his optional questions.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first two are worth 12 points each; the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 12 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Good luck.

## Section 1.

**Answer all of the following three questions.**

1. **[12 points]**

   We wish to construct an Edgeworth Box for a case in which the two agents have altruism for each other.

   Suppose Person 1 consumes $x_1$ units of good $x$ and $y_1$ units of good $y$, and suppose Person 2 consumes $x_2$ units of good $x$ and $y_2$ units of good $y$. Suppose the total amount of $x$ available to the two persons is 1 unit and suppose the total amount of $y$ available to the two persons is also 1 unit.

   (a) First suppose the utility functions of the two persons are given by

   $$U_1(x_1, y_1, U_2) = x_1 + y_1 + \tfrac{1}{2}U_2$$
   $$U_2(x_2, y_2, U_1) = x_2 + y_2 + \tfrac{1}{2}U_1 .$$

   By solving for $U_1$ as only a function of $x_1$ and $y_1$, and solving for $U_2$ as only a function of $x_2$ and $y_2$, argue that the contract curve would be the same as if the two persons were not altruistic, indeed the same as if the utility functions of the two persons were instead given by

   $$U_1(x_1, y_1) = x_1 + y_1$$
   $$U_2(x_2, y_2) = x_2 + y_2 .$$

   (b) Re-work part (a) supposing that the utility functions of the two persons are *not* given by

   $$U_1(x_1, y_1, U_2) = x_1 + y_1 + \tfrac{1}{2}U_2$$
   $$U_2(x_2, y_2, U_1) = x_2 + y_2 + \tfrac{1}{2}U_1 .$$

   but instead are given by

   $$U_1(x_1, y_1, U_2) = x_1 y_1 + \tfrac{1}{2}U_2$$
   $$U_2(x_2, y_2, U_1) = x_2 y_2 + \tfrac{1}{2}U_1 .$$

   You should obtain

   $$U_1(x_1, y_1) = 2x_1y_1 - \tfrac{2}{3}x_1 - \tfrac{2}{3}y_1 + \tfrac{2}{3}$$
   $$U_2(x_2, y_2) = 2x_2y_2 - \tfrac{2}{3}x_2 - \tfrac{2}{3}y_2 + \tfrac{2}{3} .$$

   In this case, will the outcome of the agents' behavior is affected by their altruism?

(c) The rest of this question concerns the persons of part (b). Draw an Edgeworth Box (put $x_1$ on the horizontal axis and $y_1$ on the vertical axis). In this Box, indicate the regions where:

- $\partial U_1/\partial x_1 > 0$;
- $\partial U_1/\partial x_1 < 0$;
- $\partial U_1/\partial y_1 > 0$;
- $\partial U_1/\partial y_1 < 0$.

Also in this Box, indicate the regions where:

- $\partial U_2/\partial x_2 > 0$;
- $\partial U_2/\partial x_2 < 0$;
- $\partial U_2/\partial y_2 > 0$;
- $\partial U_2/\partial y_2 < 0$.

You may reason by analogy with the results for Person 1.

(d) In each region of the Box, draw a small dot, and from this dot draw an arrow showing the direction ("northwest," "northeast," "southwest," or "southeast") of increasing utility for Person 1. Also, from each dot draw an arrow showing the dirction of increasing utility for Person 2.

(e) Considering $U_1$ as a function of $x_1$ and $y_1$, and considering $U_2$ as a function of $x_1$ and $y_1$ (*not* of $x_2$ nor $y_2$), it is trivial to show that

$$\begin{array}{ll} U_1(0,0) = \frac{2}{3} & U_2(0,0) = \frac{4}{3} \\ U_1(\frac{1}{3},\frac{1}{3}) = \frac{4}{9} & U_2(\frac{1}{3},\frac{1}{3}) = \frac{2}{3} \\ U_1(\frac{2}{3},\frac{2}{3}) = \frac{2}{3} & U_2(\frac{2}{3},\frac{2}{3}) = \frac{4}{9} \\ U_1(1,1) = \frac{4}{3} & U_2(1,1) = \frac{2}{3} \end{array}$$

so I do not want you to prove that they are true. Use these facts to argue that the only Pareto Efficient points $(x_1, y_1)$ in this two-person economy are $(0,0)$ and $(1,1)$.

(f) The conclusion for this economy is that in it, altruism leads to complete inequality as the only efficient allocation. This may be counterintuitive; the explanation can only be that the rich people are happy because they are rich, and the poor people are happy because the rich people are rich. Briefly conjecture what parameter values of the utility functions would have to be changed in order to reverse the conclusion that "altruism leads to complete inequality as the only efficient allocation." (One or two sentences will suffice here; do not work out anything mathematically, because I am asking only for a reasonable conjecture, not a proof.)

2. [12 points]
Suppose a consumer receives utility from two goods $x_1$ and $x_2$.

(a) How does this consumer's demand for good 1 change when $p_1$ changes infinitesimally?

(b) How does this consumer's demand for good 1 change when $p_2$ changes infinitesimally?

(c) Express, as a function of the change in $p_1$, how this consumer's demand for good 1 changes when: "$p_1$ and $p_2$ change simultaneously in such a way that $p_1 + p_2$ is unchanged."

3. [18 points]

**Required Question**

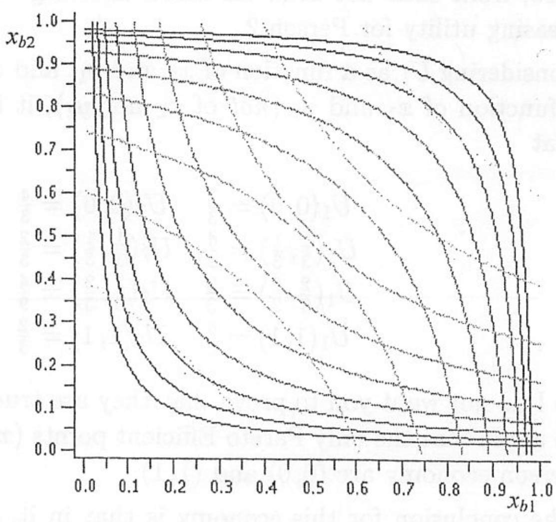
*Story One:* Barbie and Ken consume two private goods, coffee $x_1$, and muffins $x_2$. The utility functions and endowments are given as follows:

| | | |
|---|---|---|
| Barbie | $U_b = \ln(x_{b1}) + \ln(x_{b2})$ | $\omega_b = (0.8, 0.2)$, |
| Ken | $U_k = \ln(x_{k1}) + \ln(x_{k2})$ | $\omega_k = (0.2, 0.8)$. |

A feasible general equilibrium is described by $0 = \omega_{b1} + \omega_{k1} - x_{b1} - x_{k1}$ and $0 = \omega_{b2} + \omega_{k2} - x_{b2} - x_{k2}$. Barbie and Ken agree on the social welfare function,

$$W = \min(U_b, U_k).$$

The diagram below plots the indifference curves of both with respect to Barbie's consumption bundle.




(a) Identify the endowment point and the core. Find the Walrasian equilibrium from the given endowment. What is the equilibrium price vector? Illustrate your answer.

(b) Find the Pareto set (contract curve) and the social optimum. Illustrate your answer.

*Story Two:* Barbie and Ken consume a private good, coffee $x_i$, and a public good, poetry $G$. The utility functions and endowments (of coffee) are given as follows:

| | | |
|---|---|---|
| Barbie | $U_b = \ln(x_b) + \ln(G)$ | $\omega_b = 1$, |
| Ken | $U_k = \ln(x_k) + \ln(G)$ | $\omega_k = 1$. |

Both may make a contribution $g_i$ toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i.$$

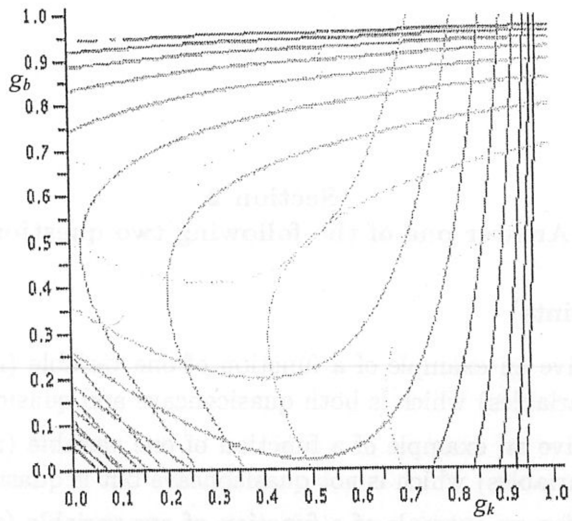
The coffee can be transformed into poetry according to the transformation function

$$0 = x_b + x_k + g_b + g_k - \omega_b - \omega_k.$$

Again, Barbie and Ken agree on the social welfare function,

$$W = \min(U_b, U_k).$$

The diagram below plots the indifference curves of both in contribution space.




(c) Find the Nash and the Lindahl equilibriums. Would both favor a move from the Nash to the Lindahl?

(d) How are the two stories similar? How do they differ? Discuss how the First Theorem of Welfare Economics applies to each?

(e) Find the Pareto set and the social optimum. Illustrate your answer.

(f) Could an allocation fail to be Pareto efficient, but still be socially optimal? Could it fail to be socially optimal, but still be Pareto efficient? Does you answer differ between the two stories?

## Section 2.

**Answer one of the following two questions.**

1. [12 points]

   (a) Give an example of a function of one variable (not two or more variables) which is both quasiconcave and quasiconvex.

   (b) Give an example of a function of one variable (not two or more variables) which is not quasiconcave but is quasiconvex.

   (c) Give an example of a function of one variable (not two or more variables) which is quasiconcave but not quasiconvex.

   (d) Give an example of a function of one variable (not two or more variables) which is quasiconcave but not concave.

   (e) Give an example of a function of one variable (not two or more variables) which is quasiconvex but not convex.

   You do not have to choose functions which are defined over the entire real line; a domain which is only part of the entire real line is acceptable.

2. [12 points]

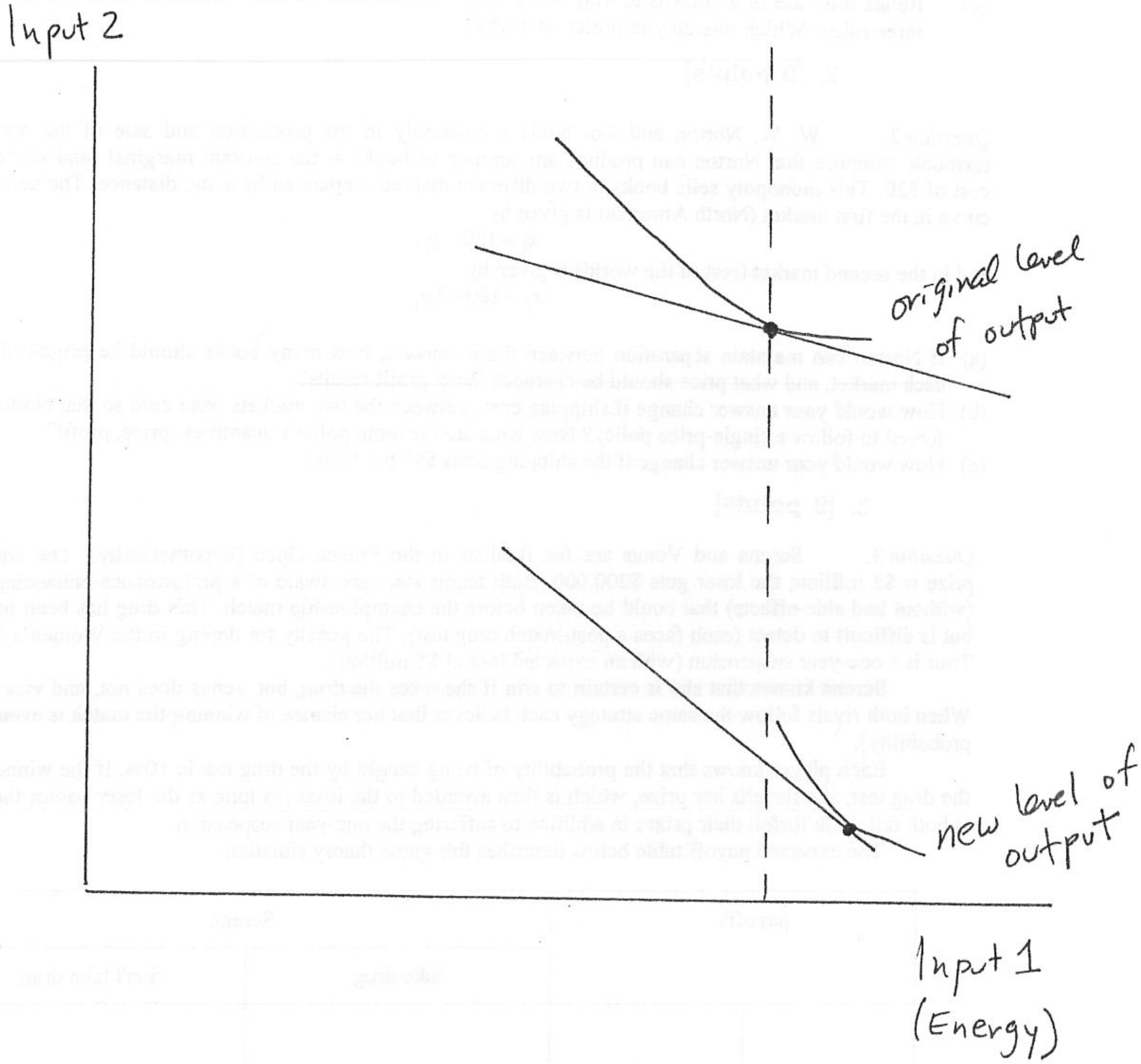
   (a) Prove Hotelling's Lemma $\mathbf{y} = \nabla_{\mathbf{p}}\, \pi(\mathbf{p})$.

   (b) Prove that the profit function $\pi(\mathbf{p})$is convex.

   (c) Refer to Figure 1 and consider the following argument:

   > Suppose the price of Input 1 (energy) rises. A competitive firm may respond to this by decreasing output. If it does, the amount of Input 1 (energy) purchased by this firm may actually rise, as illustrated in Figure 1.

   Explain why this argument is false.



Input 2
original level of output
new level of output
Input 1 (Energy)


Fig. 1

Section 3.

Answer two of the following three questions.

1. [9 points]

**Shorter Answer Questions:**
Answer two.

*Question* 1. Consider a society of 5 citizens who have rankings over three candidates (Barack, Hillary, and Dennis) as shown.

| ranking | 2 voters | 3 voters |
|---|---|---|
| first | Barack | Hillary |
| second | Hillary | Dennis |
| third | Dennis | Barack |

(a) Consider the following social decision rules:
- What is the Condorcet rule? Who is the "Condorcet winner?"
- What is the plurality rule? Who is the "plurality winner?"
- What is the Borda count rule? Who is the "Borda winner?"

(b) Which of these rules do Dasgupta and Maskin prefer? Explain.

(c) Relate this case of 5 citizens to May's and Arrow's theorems. Do these theorems favor any of these three rules? Which rule do you prefer, and why?

2. [9 points]

*Question* 2. W. W. Norton and Co. holds a monopoly in the production and sale of the Varian textbook. Suppose that Norton can produce any amount of books at the constant marginal (and average) cost of $20. This monopoly sells books in two different markets, separated by some distance. The demand curve in the first market (North America) is given by

$$x_1 = 180 - p_1,$$

and in the second market (rest of the world) is given by

$$x_2 = 180 - 3p_2.$$

(a) If Norton can maintain separation between these markets, how many books should be produced for each market, and what price should be charged? What profit results?

(b) How would your answer change if shipping costs between the two markets were zero so that Norton is forced to follow a single-price policy? Now what are the monopolist's quantities, price, profit?

(c) How would your answer change if the shipping costs $40 per book?

3. [9 points]

*Question* 3. Serena and Venus are the finalists in the French Open (hypothetically). The winner's prize is $2 million; the loser gets $200,000. Both tennis stars are aware of a performance-enhancing drug (without bad side-effects) that could be taken before the championship match. This drug has been banned, but is difficult to detect (each faces a post-match drug test). The penalty for doping in the Women's Tennis Tour is a one-year suspension (with an expected loss of $5 million).

Serena knows that she is certain to win if she takes the drug, but Venus does not, and vice versa. When both rivals follow the same strategy each believes that her chance of winning the match is even (50% probability).

Each player knows that the probability of being caught by the drug test is 10%. If the winner fails the drug test, she forfeits her prize, which is then awarded to the loser (as long as the loser passes the test). If both fail, both forfeit their prizes in addition to suffering the one-year suspension.

The expected payoff table below describes this game theory situation.

| payoffs | | Serena | |
|---|---|---|---|
| | | take drug | don't take drug |
| Venus | take drug | | |
| | don't take drug | Venus expects $0.38 million, Serena expects $1.3 million | Venus expects $1.1 million, Serena expects $1.1 million |

(a) Explain why $1.1 million is the expected payoff in the lower-right corner of the table. Explain why the numbers $0.38 and $1.3 million in the table are consistent with the scenario given above. Fill in the missing payoffs in the matrix.

(b) Only one game is played. Does either player have a dominant strategy? What is the Nash equilibrium of this game?

(c) How would the outcome change if Serena and Venus are risk averse?