Required Question
Consider a two-equation endogenous stabilization model:
\[ \pi = E\pi + x + \varepsilon, \quad \text{Phillips curve} \]
\[ \pi = \frac{1}{2}(\pi^T + E\pi + \varepsilon), \quad \text{president's policy rule} \]
where the endogenous variables are:
\( \pi = \) inflation rate,
\( x = \) output gap;
and the exogenous variables are:
\( \varepsilon = \) a random price shock,
\( E\pi = \) expected inflation,
\( \pi^T = \) the president's inflation target, where \( \pi^T > 0 \).

(a) Show that the policy rule above can be derived as a constrained optimization problem when the president’s objective function is
\[ W = -(\pi - \pi^T)^2 - x^2 \]
Use the diagram below to illustrate your answer. You should label the unlabelled lines and points, add additional explanation and include it with your answer. Explain the logic of this model.

(b) Explain how an aggregate demand and supply theory is implicit in this model.
(c) Evaluate the signs of \( \frac{\partial \pi}{\partial \pi^T} \) and \( \frac{\partial x}{\partial \pi^T} \). If possible, interpret the signs of your results. Illustrate your answer with a diagram.
(d) What is the long-run equilibrium of this model? Illustrate.
(e) Discuss whether this model can be used to explain the difference between the new-Keynesian and new-classical schools of thought.
(f) Discuss whether this model can be used to interpret economic ideology.
Optional Questions
Answer two.

Question 1. Consider a simple macroeconomy:

\[
\frac{w}{p} = F[(1-u)N], \quad F' < 0, \quad \text{labor demand}
\]

\[
Y = F[(1-u)N], \quad F' > 0, \quad \text{production function}
\]

\[
\frac{M}{p} = m(Y), \quad m_Y > 0, \quad \text{money market equilibrium}
\]

where \(Y, u\) and \(p\) are endogenous and \(w, N\) and \(M\) are exogenous. All variables are labeled as in Sargent, with \(u\) denoting the unemployment rate. \(F'\) is the marginal product of labor (possibly measured at a level of employment less than full) and \(F''\) is the second derivative of the production function with respect to employed labor.

(a) Obtain the total differential of these equations. Write this linearized system as a matrix equation, \(J dy = dx\), where \(J\) is the Jacobian matrix, \(dy\) a vector of endogenous partials and \(dx\) a vector of exogenous effects. Find the Jacobian determinant. Evaluate your result.

(b) Use the implicit function theorem to evaluate the sign of \(\frac{\partial Y}{\partial M}\). Interpret your results. Illustrate your answer with a diagram.

(c) Discuss whether this is a classical or Keynesian model. Does it exhibit neutrality? Dichotomy?

Question 2.

We economists proudly distinguish ourselves from the lower social sciences by pointing to our illustrious theoretical heritage. In the economist’s world, rational and self-interested people optimize subject to constraints. The resulting decision rules equating “marginal this” to “marginal that” lead to supplies and demands, which interact in markets to determine prices. These prices, in turn, guide the allocation of resources and the distribution of income. If not interfered with, markets tend to be highly competitive and have a strong tendency to clear by price.

Alan Blinder, “The fall and rise of Keynesian economics,” Economic Record, 1988

Using this quote as a basis, criticize any of the macroeconomic models that you have studied this year. Be specific.
Question 3. Consider a simple economy of $N$ identical worker-investor-consumers. Each is endowed with 1 unit of labor power and nominal money $m_a$. Workers offer their labor to a single firm independently of the wage rate. The single firm produces a consumption good $Y$ according to the production function

$$Y = \sqrt{(1-u)N},$$

where $u$ is the unemployment rate. Taking the wage rate and price as given, the firm maximizes profit.

In their role as investors, all workers receive equal shares of the firm’s profits,

$$\Pi = pY - w(1-u)N .$$

where $p$ is the price of the consumption good produced and $w$ is the nominal wage.

In their role as consumers, everybody allocates their budgets between consumption $c_i$ and real money balances $\left(\frac{m_i}{p}\right)$. Consumers face the budget constraint

$$\left(\frac{w + \Pi}{N}\right) + m_w = pc_i + m_i.$$ 

The consumption decision is made according to competitive assumptions; that is, consumers take $w, p$ and $\Pi$ as given.

Taxation is not permitted in this society, although positive government spending is. Equilibrium in the goods market is described by

$$Y = C + G = \sum_{i=1}^{S} c_i + G = N \left( \frac{1}{2} \left( \frac{Y}{N} + \frac{m_w}{p} \right) \right) + G.$$ 

The government controls the supply of nominal money; equilibrium in the money market is described by

$$\frac{M}{p} = \sum_{i=1}^{S} \frac{m_i}{p} = N \left( \frac{1}{2} \left( \frac{Y}{N} + \frac{m_w}{p} \right) \right).$$

Assume that $p, u, Y, C$ and $G$ are endogenous, while $w, M, m_w$ and $N$ are exogenous.

(a) Find the aggregate supply curve $p = S(Y)$. Show that this equation can be reinterpreted as a Phillips curve. Illustrate your answer.

(b) Given that $w=1/2, M=100, m_w=1$, and $N=100$, find equilibrium values for $p, u, Y, C$ and $G$. Illustrate your answer.

(c) Explain why fiscal and monetary policies are not independent in this economy, and why monetary policy is effective.
1. Kaldor’s trade cycle:
   The system of differential equations of the Kaldor model is comprised of partial adjustment to
   investment–savings imbalances (with speed $0 < \alpha < 1$) in the goods market and the capital accu-
   mulation rule (with depreciation $0 < \delta < 1$). Assume that savings increase linearly and investment
   non–linearly with $Y$, so that the Keynesian stability condition is violated (satisfied) at normal (low
   or high) levels of output. Briefly motivate both differential equations. Derive Jacobian, isoclines,
   and present the phase diagram with arrows of motion. State further assumptions as necessary, and
   briefly explain.

2. Goodwin dynamics:
   Discuss the significance of the Goodwin model: its structure, strengths and weaknesses, as well as
   how the heterodox macroeconomic literature has build on it. (Write a well structured, clear essay;
   pay attention to theory, empirics as well as formal models without presenting details of the latter.)

3. Kaleckian accumulation and capital intensity:
   (a) Set up a Kaleckian model in the rate of accumulation $g$ and capital intensity $k$ based on the
   following assumptions: (1) the actual rate of accumulation $g$ slowly adjusts to the desired
   rate of accumulation $f$, which is a positive function of $u = Y/K$, a positive function of $\pi$, and
   a negative function of $e = L/N$; (2) the mark–up is fixed; (3) in the short run demand
   clears the goods market given $g$; (4) the law of evolution of capital intensity follows from its
   definition, $k = K/N$; and (5) labor productivity is constant and normalized to 1. Make further
   assumptions as needed; discuss.

   (b) Show $J$ and $J^*$. Discuss the sign pattern. Draw a phase diagram. (For the Jacobians, put $g$
   in the first row; for the phase diagram, put $g$ on the vertical axis.) Briefly explain.

   (c) Show the linearized system of differential equations around $g^* = 0.1$ and $k^* = 1$ with
   $f_e = -0.15, f_u = 0.15, s_\pi = 0.9, \pi = 1/3$ and a speed of adjustment $\alpha = 0.1$. Calculate roots, trace,
   determinant and isoclines, and comment on the systems behavior. Is this parameterization
   reasonable?

   (d) Now abstract from capital intensity $k$. What is the effect of redistribution on capital accumu-
   lation? Differentiate around $\dot{g} = 0$ to discuss the sign of the growth regime. Present the phase
   diagram and discuss.